Imperative Logic

<table>
<thead>
<tr>
<th>Indicative</th>
<th>Imperative</th>
</tr>
</thead>
<tbody>
<tr>
<td>(You’re doing A.)</td>
<td>(Do A.)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Au</td>
<td>Au</td>
</tr>
</tbody>
</table>

1. Any underlined capital letter is a wff.
2. The result of writing a capital letter and then one or more small letters, one small letter of which is underlined, is a wff.
Don’t do A = \sim A

Do A and B = (A \cdot B)

Do A or B = (A \lor B)

Don’t do either A or B = \sim (A \lor B)

Don’t both do A and do B = \sim (A \cdot B)

Don’t combine doing A with doing B = \sim (A \cdot B)

Don’t combine doing A with not doing B = \sim (A \cdot \sim B)

Don’t do A without doing B = \sim (A \cdot \sim B)
You’re doing A and you’re doing B = (A ⋅ B)
You’re doing A, but do B = (A ⋅ B)
Do A and B = (A ⋅ B)

If you’re doing A, then you’re doing B = (A ⊃ B)
If you (in fact) are doing A, then do B = (A ⊃ B)
Do A, only if you (in fact) are doing B = (A ⊃ B)

If you (in fact) are doing A, then don’t do B = (A ⊃ ¬B)
Don’t combine doing A with doing B = ¬(A ⋅ B)
X, do (or be) A = Ax
X, do A to Y = Axy

Everyone does A = (x)Ax
Let everyone do A = (x)Ax

Let everyone who (in fact) is doing A do B = (x)(Ax ⊃ Bx)
Let someone who (in fact) is doing A do B = (∃x)(Ax • Bx)
Let someone both do A and do B = (∃x)(Ax • Bx)
Imperative Arguments

If the cocoa is about to boil, remove it from the heat.

The cocoa is about to boil.

∴ Remove it from the heat.

(B ⊨ R)  Valid

B

∴ R

• Use the same inference rules as before; but treat “A” and “A” as different wffs.
• An argument is VALID if it is inconsistent to join the premises with the contradictory of the conclusion.
• Alternatively, VALID = if the premises are correct (“1”) then so must be the conclusion.
Don’t combine accelerating with braking.
You’re accelerating.
∴ Don’t brake.

* 1 \( \sim(A^\circ \cdot B^1) = 1 \) Invalid
2 \( A^1 = 1 \)
   \[ \therefore \sim B^1 = 0 \]
3 \( \text{asm: } B \)
4 \( \therefore \sim A \) \{from 1 and 3\}

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On our refutation:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>1</td>
</tr>
<tr>
<td>( \sim A )</td>
<td>0</td>
</tr>
<tr>
<td>( B )</td>
<td>1</td>
</tr>
</tbody>
</table>

This is consistent:

You’re accelerating.
Don’t accelerate.
Instead, brake.
Don’t combine accelerating with braking. \( \neg (A \cdot B) \) Invalid
You’re accelerating. \( A \)
\[ \therefore \text{Don’t brake.} \]
\( \therefore \neg B \)

If you’re accelerating, then don’t brake. \( (A \supset \neg B) \) Valid
You’re accelerating. \( A \)
\[ \therefore \text{Don’t brake.} \]
\[ \therefore \neg B \]

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\( (A \supset \neg B) \) = If you do A, then don’t believe that A is wrong.
\( (B \supset \neg A) \) = If you believe that A is wrong, then don’t do A.
\( \neg (B \cdot A) \) = Don’t combine believing that A is wrong with doing A.
# Deontic Logic

<table>
<thead>
<tr>
<th>Indicative</th>
<th>Imperative</th>
<th>Deontic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(You’re doing A.)</td>
<td>(Do A.)</td>
<td>(You ought to do A.)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>OA</td>
</tr>
<tr>
<td>Au</td>
<td>Au</td>
<td>OAu</td>
</tr>
</tbody>
</table>

3. The result of writing “O” or “R,” and then an imperative wff, is a deontic wff.
OA = It’s obligatory that A.
OA_X = X ought to do A.
OA_{xy} = X ought to do A to Y.

RA = It’s permissible that A.
RA_X = It’s all right for X to do A.
RA_{xy} = It’s all right for X to do A to Y.

Act A is wrong = \sim RA = Act A isn’t all right.
= O\sim A = Act A ought not to be done.
It ought to be that $A$ and $B$  \[= O(A \cdot B)\]
It’s all right that $A$ or $B$  \[= R(A \lor B)\]

If you do $A$, then you ought not to do $B$  \[= (A \supset O \neg B)\]
You ought not to combine doing $A$ with doing $B$  \[= O \neg (A \cdot B)\]

It’s obligatory that everyone do $A$  \[= O(x)A_x\]
It isn’t obligatory that everyone do $A$  \[= \neg O(x)A_x\]
It’s obligatory that not everyone do $A$  \[= O \neg (x)A_x\]
It’s obligatory that everyone refrain from doing $A$  \[= O(x)\neg A_x\]
It’s obligatory that someone answer the phone  
\[= \text{O}(\exists x)Ax\] 

There’s someone who has the obligation to answer the phone  
\[= (\exists x)\text{O}Ax\] 

It’s obligatory that some who kill repent  
\[= \text{O}(\exists x)(Kx \cdot Rx)\] 

It’s obligatory that some kill who repent  
\[= \text{O}(\exists x)(Kx \cdot Rx)\] 

It’s obligatory that some both kill and repent  
\[= \text{O}(\exists x)(Kx \cdot Rx)\]
Deontic Proofs

- A *world prefix* is a string of zero or more instances of “W” or “D.”

- A *possible world* is a consistent and complete set of indicatives and imperatives.

- A *deontic world* is a possible world in which the indicative statements are all true and the imperatives prescribe some jointly permissible combination of actions.

- “OA” is true if and only if “A” is in all deontic worlds.

- “RA” is true if and only if “A” is in some deontic worlds.
Suppose that these indicatives are all true:

- I have an 8 am class.
- I ought to get up before 7 am.
- It would be permissible for me to get up at 6:45 am.
- It would be permissible for me to get up at 6:30 am.

Then deontic worlds D and DD might contain these, in addition to the indicatives listed above:

<table>
<thead>
<tr>
<th></th>
<th>Get up before 7 am.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Get up at 6:45 am.</td>
</tr>
<tr>
<td>DD</td>
<td>Get up before 7 am.</td>
</tr>
<tr>
<td></td>
<td>Get up at 6:30 am.</td>
</tr>
</tbody>
</table>
### Deontic Inference Rules

<table>
<thead>
<tr>
<th>First reverse squiggles</th>
<th>(~\text{OA} \rightarrow R\sim A)</th>
<th>(~\text{RA} \rightarrow O\sim A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>and drop R’s;</td>
<td>(\text{RA} \rightarrow D \bowtie A), use a <em>new</em> string of D’s</td>
<td></td>
</tr>
<tr>
<td>lasty, drop O’s.</td>
<td>(\text{OA} \rightarrow D \bowtie A), use a blank or any string of D’s</td>
<td>Don’t star</td>
</tr>
</tbody>
</table>
Indicative transfer

D : A → A,
the world prefixes of the
derived and deriving steps
must be identical except
that one ends in one or
more additional D’s

We can transfer
indicatives freely
between a deontic
world and whatever
world it depends on.

Kant’s Law

"Ought” implies “can”:
“You ought to do A” entails
“It’s possible for you to do A.”
Hare’s Law: An “ought” entails the corresponding imperative.

Kant’s Law: “Ought” implies “can.”

Hume’s Law: You can’t deduce an “ought” from an “is.”

Poincaré’s Law: You can’t deduce an imperative from an “is.”
* 1 \[ R(A \cdot B) \]
\[ \therefore RA \]

* 2 \[ \text{asm: } \sim RA \]

3 \[ \therefore O \sim A \] \{from 2\} \hspace{2cm} \leftarrow \text{reverse squiggles}

* 4 \[ D \therefore (A \cdot B) \] \{from 1\} \hspace{2cm} \leftarrow \text{drop “R”}

5 \[ D \therefore A \] \{from 4\}

6 \[ D \therefore B \] \{from 4\}

7 \[ D \therefore \sim A \] \{from 3\} \hspace{2cm} \leftarrow \text{drop “O”}

8 \[ \therefore RA \] \{from 2; 5 contradicts 7\}

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1. Reverse squiggles.
2. Drop each initial “R,” using a new deontic world each time.
3. Lastly, drop each initial “O” once for each old deontic world.
   (Never use a new deontic world when you drop “O.”)
\[ \vdash \Box (O(A \cdot B) \supset OA) \quad \text{Valid} \]

* 1 \hspace{1cm} \text{asm: } \neg \Box (O(A \cdot B) \supset OA)

* 2 \hspace{1cm} \therefore \Diamond \neg (O(A \cdot B) \supset OA) \quad \{ \text{from 1} \}

* 3 \hspace{1cm} W \vdash \neg (O(A \cdot B) \supset OA) \quad \{ \text{from 2} \}

4 \hspace{1cm} W \vdash O(A \cdot B) \quad \{ \text{from 3} \}

* 5 \hspace{1cm} W \vdash \neg OA \quad \{ \text{from 3} \}

* 6 \hspace{1cm} W \vdash R\neg A \quad \{ \text{from 5} \}

7 \hspace{1cm} WD \vdash \neg A \quad \{ \text{from 6} \}

8 \hspace{1cm} WD \vdash (A \cdot B) \quad \{ \text{from 4} \}

9 \hspace{1cm} WD \vdash A \quad \{ \text{from 8} \}

10 \hspace{1cm} \vdash \Box (O(A \cdot B) \supset OA) \quad \{ \text{from 1; 7 contradicts 9} \}