Teacher's Manual

for *Introduction to Logic*

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http://www.harryhiker.com/lc

London and New York

Using the Textbook

My *Introduction to Logic* is a comprehensive introduction. It covers:

- syllogisms;
- propositional and quantificational logic;
- modal, deontic, and belief logic;
- the formalization of an ethical theory;
- metalogic; and
- induction, meaning/definitions, and fallacies/argumentation.

Because of its broad scope, the book is well suited for either basic or intermediate courses in logic; the end of Chapter 1 of the book talks about which chapters to use for which type of course, and which chapters presume which other chapters.

The book grew out of my experience teaching two types of logic course. The first is a basic "baby logic" course, intended for general undergraduate students. This is what I cover (where each class period is 50 minutes):

- Chapters 1 and 2: Introduction and syllogisms (7 class periods + a full-period test). I assign LogiCola sets A (EM, ET, HM, & HT) and B (H, S, E, D, C, F, & I).
- Chapter 3: Basic propositional logic (7 class periods + a full-period test). I assign LogiCola sets C (EM, ET, HM, & HT); D (TE, TM, TH, UE, UM, UH, FE, FM, FH, AE, & AM); E (S, E, F, & I); and F (SE, SH, IE, IH, CE, & CH).
- Chapter 4: Propositional proofs (7 class periods + a full-period test). I assign LogiCola sets F (TE & TH) and G (EV, EI, EC, HV, HI, HC, & MC).
- Chapter 7: Basic modal logic (7 class periods + a full-period test). I assign LogiCola sets J (BM & BT) and K (V, I, & C). The last three class periods are split; the first part of the period is on modal logic while the second is on informal fallacies.
- Chapters 5 and 15 (Sections 15.1 & 15.2): Basic quantificational logic and informal fallacies (7 class periods + a final exam – which is 3/7 on the new material and 4/7 on pervious material). I assign LogiCola sets R; H (EM, ET, HM, & HT); and I (EV, EI, EC, HC, & MC). The first two class periods are split; the first part is on informal fallacies while the second is on quantificational logic. The last class is a review.

I also teach a more advanced "Symbolic Logic" course. This is intended for philosophy majors and minors and graduate students – and others who want a more demanding logic course. Since most have had no previous logic, I start from the beginning but move quickly. This is what I cover (where again each class period is 50 minutes):

- Chapters 1 and 3: Introduction and basic propositional logic (6 class periods + a half-period quiz; the first half of the quiz period introduces the material for the next part). I assign LogiCola sets C (EM, ET, HM, & HT); D (TE, TM, TH, UE, UM, UH, FE, FM, FH, AE, & AM); E (S, E, F, & I); and F (SE, SH, IE, IH, CE, & CH).

- Chapters 4 and 12 (Sections 12.1 to 12.4): Propositional proofs and metalogic (the half-period mentioned above + 4 class periods + a half-period quiz; the first half of the quiz period introduces the material for the next part). I assign LogiCola sets F (TE & TH) and G (EV, EI, EC, HV, HI, HC, & MC).
- Chapter 5: Basic quantificational logic (the half-period mentioned above + 5 class periods + a half-period quiz; the first half of the quiz period introduces the material for the next part). I assign LogiCola sets H (EM, ET, HM, & HT) and I (EV, EI, EC, HC, & MC).
- Chapter 6: Relations and identity (the half-period mentioned above + 4 class periods + a half-period quiz; the first half of the quiz period introduces the material for the next part). I assign LogiCola sets H (IM, IT, RM, & RT) and I (DC, RC, & BC).
- Chapters 7 and 8: Modal logic (the half-period mentioned above + 5 class periods + a half-period quiz; the last half of the last class period and the first half of the quiz period introduces the material for the next part). I assign LogiCola sets J (BM, BT, QM, & QT) and K (V, I, C, G, & Q).
- Chapter 9: Deontic logic (the two half-periods mentioned above + 3 class periods + a half-period quiz; the first half of the quiz period introduces the material for the next part). I assign LogiCola sets L (IM, IT, DM, & DT) and M (I, D, & M).
- Chapters 10 and 11: Belief logic and a formalized ethical theory (the half-period mentioned above + 6 class periods + a comprehensive final exam that more heavily weights the material from Chapters 10 and 11); if I have time, I also do Section 12.7 on Gödel's theorem. I assign LogiCola sets N (BM, BT, WM, WT, RM, & RT) and O (B, W, R, & M).

If I get behind, I skip or cover quickly some sections that won't be used much further on (for example, 6.5, 8.1, 8.4, and 10.7). While this course covers much material, my students don't complain about the workload; most rate my course as being "of average difficulty."

I advise against trying to cover the whole book in a single course. Since the book has much material, you'll have to pick what to use. What I use, as sketched above, is given as an example. You'll likely want to cover a different selection of materials or use a different order.[1]

In deciding which chapters to teach, I suggest that you consider questions like these:

- "How bright are your students?" Teach relations and identity only if your students are very bright. Even basic quantification and modal logic may be too hard for some groups.
- "What areas connect with the interests of your students?" Science majors have a special interest in induction, communications majors in informal fallacies, math majors in quantification, philosophy majors in modal logic and meaning (and the end of chapter 15), and so on. Students in practical fields (like business) often prefer the early formal

[1] My explanations here assume that the book is the main or sole textbook for a one-semester (or one-quarter) course. You could cover the whole book in a two-semester course. Or, alternatively, you could use just a few chapters of the book in a specialized course on topics like "modal logic," "deontic and epistemic logic," or "ethics and logic" (this last one might also use my *Formal Ethics* or Chapters 7–9 of my *Ethics: A Contemporary Introduction*).

chapters and their direct application to everyday arguments.

- "What areas do you most enjoy?" Other things being equal, you'll do a better job if you teach the areas most important to you – whether this be mostly formal, mostly informal, or a mix of both.

You'll need to experiment and see what works for you and for your mix of students.

Sequence is another issue. My basic course starts with syllogisms – an easy system with many applications. Then I move to propositional logic. I do modal logic before quantification, since modal logic is easier and applies to more interesting arguments. I do informal logic last, since I like students to have a good grounding in what makes for a valid argument before they do informal logic.

Some teachers prefer other sequences. Some use syllogisms to ease the transition between propositional and quantificational logic. Others start with informal logic and later move into the more technical formal logic. The textbook allows all these approaches. You might experiment with various sequences.

The text uses simpler methods for testing arguments than the standard approaches. Students find my star test for syllogisms and my method of doing formal proofs easy to learn. Also, the text is simply written. For these reasons, you may be able to cover more material than you would have thought; keep this in mind as you plan your course. Since some of my methods are unconventional, you should first master these methods yourself; the computer instructional software gives an easy way to do this.

Your main role in class is to go through problems with your students, giving explanations and clarifications as you go along. Focus on *rules-and-examples* taken together. The explanations in the book may seem clear to you; but most students need to see "how to do it" over and over before they get the point. Students vary greatly in their aptitude for logic. Some pick it up quickly and hardly need the teacher; others find logic difficult and need individual tutoring. Most students are in the middle. Most students find logic very enjoyable.

The Web sites (see the Web addresses on the cover page of this manual) have downloadable classroom slides in Adobe Acrobat format for many of the chapters. If your classrooms have a computer connected to an projector, you can project these slides directly from the computer. An alternative is to print out the pages and use them with a standard overhead projector.

I give many tests: 4 full-period test + a final exam in my basic logic course, and 6 short (25 minute) quizzes + a final exam in my more advanced course. Breaking the material into smaller bunches makes it easier to learn; and some students don't get serious until there's a test. My test questions are like the problems in the book, except that I use multiple-choice or short-answer questions for Chapters 11 and 12. The Web sites (see the Web addresses on the cover page of this manual) have sample tests. In my basic logic course, each test is three pages long; to make cheating harder, I staple the three pages in random order. I suggest that you time how long it takes you to do a test that you'll give to your class; a test that I can do in 9 or 10 minutes is about the right length for my class to do in a 50 minute period.

I record LogiCola scores whenever I give a test. I bring my laptop computer to the classroom and record scores at the beginning – which takes about five minutes. I use the LogiCola scores as a bonus or penalty to be added to the student's score on the written test.

Those are my general comments. Let me talk about individual chapters.

Chapter 1. Introduction

This chapter is very easy. In class I give a brief explanation (with entertaining examples) of the key ideas: argument, validity, and soundness. I don't spend much time on this.

I give my baby logic class a pretest the first day, before they read Chapter 1. The test has 10 multiple-choice problems. The students do the test and then correct it themselves (the answer key is on the second page); this takes just a few minutes. Then I go through the first five problems; I ask the students why a particular answer would be wrong – and the students tend to give good answers. The pretest gets them interested in logic right away, gives them an idea of what logic is, and lets them see that there are good reasons for saying that something does or does not follow from a set of premises. If you want to give the pretest to your class, download it from the Web sites (see the Web addresses on the cover page of this manual) and make copies for your students.

The pretest and Chapter 1 focus on clearly stated arguments. Most books instead begin with twisted arguments (where it's hard to identify the premises and conclusion). In my book, twisted arguments come later, in Sections 2.7 and 3.9. I think it's better to move from the simple to the complex.

In your opening pep-talk, emphasize the importance of keeping up with the work. Some students do most of their studying just before an exam; then they cram. In logic, only the bright ones can get away with this. Logic is cumulative: one thing builds an another. Students who get a few steps behind can become hopelessly lost. In spite of your warnings, you'll have to be available to help out students who out of laziness or sickness fall behind.

I strongly encourage you to have your students do homework using the LogiCola computer program. LogiCola isn't a gimmick; it will make a huge difference in how well your students learn logic. The next chapter of this teacher's manual explains how to use LogiCola in your course. If you use the program, you'll want to talk about it at the beginning. I like to bring in my laptop and give a little demonstration; however, this may not be needed – since the program is easy to use and students are computer savvy these days.

You may also want to give your students flashcards; these are downloadable from the Web sites (see the Web addresses on the cover page of this manual) and you can have your copy center make copies on heavy paper. The flashcards are helpful in learning translations and inference rules. Since my students now do most of their homework on computer, they use the flashcards less than before; but most still use them and find them helpful. Students can use flashcards at odd moments when they don't have a computer handy.

Chapter 2. Syllogistic Logic

This chapter is pretty easy. Most students pick up the star test quickly (although some are confused at first on what to star). Soon most of them make almost no mistakes on testing arguments in symbols. You'll find the star test a pleasure to teach, as compared with other ways to test syllogisms. Students find the first set of English arguments easy, although they may be confused on a few translations; stress the importance of thinking out the arguments intuitively before doing the star test. The deriving-conclusions exercise is somewhat harder, as is the section on idioms. The most difficult sections, according to my students, are the ones on Venn diagrams and on idiomatic arguments (and these sections may be skipped if your students are on the slow side); students need help and encouragement on these.

The book has an abundance of problems – which can be used in different ways. In class, I typically do a couple of problems on the board (explaining how to do them as I go), give them a few to do in class (working them out on the board after they finish), and then give them a few more to do for homework (going through them the following class). Many exercise sections have a lot more problems than you'd want to cover in a given semester.

One of the strong features of my book is that the exercises tend to use important arguments – many on philosophical issues. This helps you, the teacher, show the relevance of logic in clarifying our reasoning. Occasionally spend some time on the content of the arguments. Tell the class about the context and wider significance of an argument. Ask them what premises are controversial and how they might defend or attack them. Refer to informal considerations (for example, inductive backing, definitions, or fallacies) when suitable.

Chapter 3. Basic Propositional Logic

This chapter is easy and students have little difficulty with most of it. While there are many things to learn, most of it can be covered quickly.

The inference rules (S- and I-rules) are easy for a few students and hard for many. Drill the class by giving them premises and asking them what follows using the rules. I have some standard examples (such as "If you're in Chicago then you're in Illinois – but you're in Illinois – so …") that I use to help their intuitions on valid and invalid forms. Examples with many negatives can be confusing. Students need to have a good grasp of these rules before starting formal proofs in the next chapter; otherwise they'll struggle with the proofs.

Most logicians adopt various conventions for dropping parentheses. I keep all parentheses – since explaining parentheses-dropping conventions takes up as much time as the conventions save. And many things go more smoothly if we don't drop parentheses. For example, we can use a simple rule for translating "both" as "("; so "not both" is "∼(" while "both not" is "(∼." And in doing formal proofs there's less confusion about assuming the opposite of the conclusion. You don't have to remind students that, since "P ⊃ Q" is really "(P ⊃ Q)," the contradictory of "P ⊃ Q" is "∼(P ⊃ Q)." Also, you use actual wffs and not just abbreviations for these.

Chapter 4. Propositional Proofs

This chapter is harder than the previous ones, although not as hard as the following chapters. Most students pick up the proof method easily after they've seen the teacher work out and explain various examples. Those who don't know the inference rules from the last chapter will be lost and will need to go back and learn the rules. Multiple assumption proofs are tricky at first; make sure that you understand them yourself. Spend time in class doing problems and answering questions. Soon most students get very proficient at proofs. More students get 100's on my propositional proofs test than on any other test; typically about 40 percent of the class gets 100's.

Tell your students how you want them to do proofs. While the book gives justifications for the various steps – like "{from 3 and 6}" – I make justifications optional; omitting justifications makes proofs much easier to do. On a test, I can easily tell where a step is from; while doing proofs on the board, I show where things are from through words or gestures. A few of my students include justifications anyway, even though they're optional. You, however, may want to require justifications; you may even require that students give the inference rule – perhaps saying things like "{from 3 and 6 by the I-rules}" or "{... by I-5}" or "{... by I-if}" or "{... by MP}."

I also make stars optional; but I use them when working out a problem in class. Many of my students use stars, since it gives them a guide on what to do next; but many omit them.

You should say whether you want your students to keep strictly to the S- and I-rules in deriving steps. I have students follow these rules until they're comfortable with proofs. When students are sure of themselves, they can use any step whose validity is intuitively clear to them and to their teacher. Since it's safer to follow the rules, most of my students do this.

If you're more familiar with Copi-style proofs or with truth trees, you might want to study Section 4.7, which compares these methods with mine.

Chapter 5. Basic Quantificational Logic

This chapter is harder than the previous ones. Students find the translations difficult; it's good to spend some time on the translation rules and then review as when you do the English arguments. Proofs present less of a problem. But you have to remind students to drop only initial quantifiers and to use new constants when dropping existential quantifiers. And you'll need to help students to evaluate the truth of the premises and conclusion for invalid arguments.

Chapter 6. Relations and Identity

This is one of the most difficult chapters in the book for students, with relations causing more problems than identity. Students need help and encouragement on relational translations. Relational proofs also are difficult, since they tend to be more complex and less mechanical than other proofs. If you run short on time, you could omit Section 6.5 on definite descriptions. I refer to this material in Section 8.4 (on sophisticated quantified modal logic) – which you also could omit if you're running short on time.

Chapter 7. Basic Modal Logic

Students find modal logic easier than quantificational logic, despite the similarity in structure. Translations aren't too difficult; but you'll need to explain the ambiguous forms a couple of times. Students find proofs tricky at first, until it clicks in their mind what they're supposed to do; you'll have to emphasize that they can drop only initial quantifiers and have to use a new world when dropping a diamond. And you'll have to explain refutations. The ambiguous arguments are fun to elaborate on – especially the ones about skepticism and predestination (examples 8 and 14 in Section 7.3b).

Chapter 8. Further Modal Systems

The naïve version of quantified modal logic (Sections 8.2 and 8.3) is moderately challenging and brings up some interesting philosophical controversies and arguments. The rest of the chapter is more difficult and not needed for further sections of the book; these sections could be omitted if you are running short on time or if your students find the material too difficult.

Chapter 9. Deontic and Imperative Logic

This chapter is quite easy – and students find it very interesting.

Chapter 10. Belief Logic

This chapter is difficult, especially the complex symbolizations. You'll have to point out how small differences in underlining or the placement of ":" can make a big difference to the meaning of a formula. The belief worlds and belief inference rules are less intuitive than comparable ideas of other systems. Students like the philosophical content.

Chapter 11. A Formalized Ethical Theory

This chapter starts fairly easy but gets very difficult toward the end. I stress the main features of the formalization and don't hold students responsible for the details. I run through the long proof at the end step-by-step, emphasizing to students how much of it rests on what they already know. Students like the philosophical content.

Chapter 12. Metalogic

While the beginning of this chapter is fairly easy, students find the completeness proof difficult. The section on Gödel's theorem is difficult, but students find it fascinating.

Chapter 13. Inductive Reasoning

While this chapter is long (the longest in the book), it's only moderately difficult. Many students like the more philosophical sections (13.3, 13.6, and 13.9). The exercise about how to verify scientific theories (Section 13.8a) is challenging.

Chapter 14. Meaning and Definitions

The early sections here are easy, and the later ones more difficult. Students enjoy the problems on cultural relativism, especially since many of them are going through a relativistic phase in their own thinking. Work through a few of the exercises on positivism, pragmatism, analytic/synthetic, and *a-priori/a-posteriori* before assigning the exercises (Sections 14.5a, 14.7a, and 14.8a); many won't catch on unless you first do a couple of examples with them. The exercise on making distinctions (Section 14.6a) is challenging and very valuable; I like to assign five of these at a time, and then later make a composite-answer for the class.

Chapter 15. Fallacies and Argumentation

Fallacy-identification isn't a precise art. In judging answers, you often have to bend a little on what counts as a correct answer; but you don't want to bend so much that just anything goes. Some students prefer the precision of formal logic.

Sections 15.4 and 15.5 integrate formal and informal concerns and tie the book together. While the book doesn't include exercises for these sections, you could pass out some passages for analysis. If you do this, use easy passages. A skilled logician sometimes requires several hours of hard work to extract a clear argument from a confused passage; don't give your students passages to analyze that would strain even your powers.

I've done independent study courses along the lines of Section 15.5 with small groups of two to four bright students, mostly philosophy majors, all of whom had had me in logic. The independent study course followed this format. Each week individual students would take some philosophical passage that they're reading (perhaps for a course). They would put the arguments in strict form and evaluate them (validity, truth of premises, ambiguities, etc.); they would write this out, add a photocopy of the original passage, and distribute all this to me and to the rest of the group. Then we'd get together to talk about their analyses and about the philosophical issues involved. The students found this hard work but very valuable.

Using the LogiCola Software

LogiCola (LC) is a computer program to help students learn logic. LC generates homework problems, gives feedback on answers, and records progress. Most of the exercises in the book have corresponding LogiCola computer exercises. There are LC versions for Windows, DOS, and Macintosh. The book's appendix has further information on the program: how to download it for free from the Web, how to start it, how to do exercises, and which exercises go with which chapters. Read this appendix and try the program.

I designed LogiCola to supplement classroom activity and to be used for homework. You don't have to use LC if you use the textbook. But there are two main benefits in doing so: (1) your students will learn logic better, and (2) you'll have less work to do.

If you use LogiCola, your classroom activity needn't change. But your students will do most of their homework on a computer, instead of on paper. This has major advantages, as we can see from this comparison:

Doing homework on paper	*Doing homework on LogiCola*
The paper won't talk back to your students. It won't tell them if they're doing the problems right or wrong. It won't give them suggestions. And it won't work out examples, even if students need this in order to get started.	LogiCola will talk back to your students. It'll tell them immediately if they're doing the problems right or wrong. It'll give them suggestions. And it'll work out examples, if students need this in order to get started.
The students will all get the same problems to do. So they can pass around their papers and share the answers.	LogiCola will give each of your students different problems. So they will only share hints on how to do the problems.
Students will get the corrected paper back, at best, a couple of days after doing the problems. Only then will they find out what they were doing wrong.	LogiCola's immediate response motivates students and makes learning more fun – like playing a video game. Homework doesn't have to be boring.

The traditional method of having students do homework on paper is slow and less effective. LogiCola is a better tool for learning logic. My students attest to this and give LogiCola very high ratings on course evaluations. Students enjoy the program and learn more effectively.

I noticed a big jump in test scores when I started using the program in Spring 1988. I kept careful records of test scores for the last seven sections of basic logic that I taught before using the program – and the first six sections that I taught since using the program. The "before" and "after" groups each had about 200 students – all in my PL 274 at Loyola University of Chicago (where I was teaching then). The groups and my teaching methods were very similar – except that the "after" group used LogiCola and averaged about a grade better (+7.6%) on comparable tests on the same material. Here's a chart summarizing test averages:

Tests >>	#1	#2	#3	#4	#5	Average
Before LC	77.7	82.5	77.6	73.4	76.0	77.44
After LC	84.8	90.7	84.4	83.6	81.7	85.04
Difference	+7.1	+8.2	+6.8	+10.2	+5.7	+7.60

The five tests were on syllogisms, basic propositional logic, propositional proofs, modal logic, and the comprehensive final exam; page 2 of this manual has further information on the tests. The "before" and "after" information each covers about 1000 tests (5 tests each for 200 students). The LogiCola program that my students used in the late 1980's was the early DOS version and rather primitive compared with the current version. Since the late 1980's, scores on written tests have continued to climb; but, since I've changed schools and made other changes in my course, the comparison of test scores isn't as meaningful.

Your students too will likely learn better with LogiCola. In addition, you'll have less work to do. If you have students do homework on paper, you have to correct the papers; this is boring and takes much time. Or you can just go through the problems in class; but then many students won't do the problems. If you use LogiCola, the program itself will correct the problems. When students complete an exercise at a given level of proficiency, this fact records on the disk. At the end of a chapter, you record scores using the score processor program; it takes about 12 minutes to process scores from 60 students. The computer generates a class roll listing all the scores and the resulting bonus or penalty points. I add the latter points to the scores for the corresponding written test.

This is what I say in my syllabus[1] about LogiCola and how it enters into grading:

> You'll do much of your homework on computer, using the LogiCola program (which you'll each get on a floppy disk). Turn in your disk with the assigned exercises done when you take the corresponding written quiz; I won't accept scores after I return the quiz. Try to do the exercises at an average level of 7 or higher (levels go from 1 to 9).
>
> Your exercise scores add a bonus or penalty to your exam score. Let's say your average level (dropping fractions) is N. You get a +1 bonus for each number N is above 7; for example, you get a +2 bonus if N=9. You get a -1 penalty for each number N is below 7; so you get a -3 penalty if N=4. If you fake scores electronically, your course grade will be lowered by one grade.

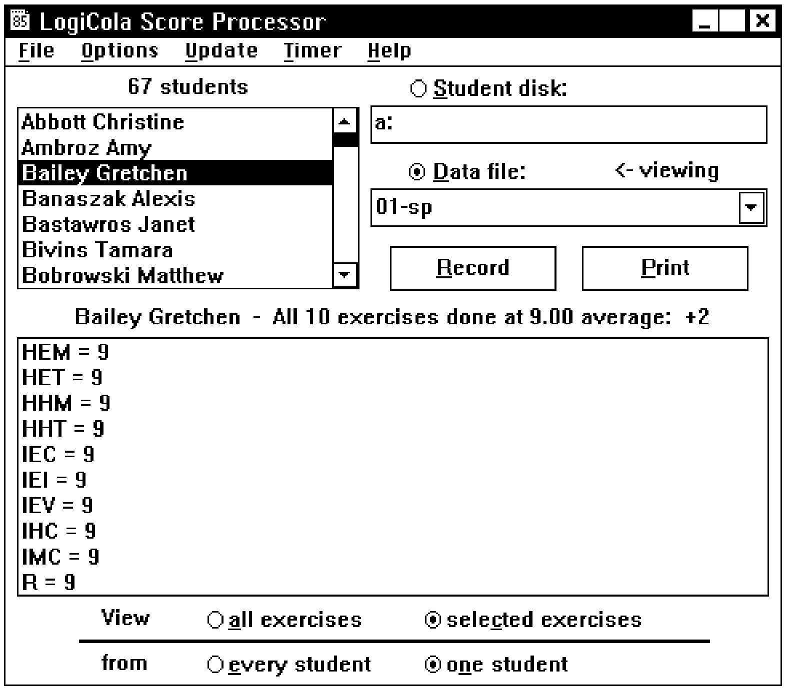
Most students do all the exercises at level 9 and thus get the +2 point bonus.

Using LogiCola requires that you (or someone else) do two computer chores: (1) record scores, and (2) give out disks with the program. The time here is small compared with the time you save in grading homework.

[1] To see my syllabus, go to http://www.harryhiker.com/courses.htm#L on the Web.

Processing Scores

You can download the score processor program from the same Web sites from which you download LogiCola itself (see the book's appendix on "LogiCola Software"). The score processor (MC-SCORE.EXE) is easy to use and looks like this:



Under "Student disk" put its location (usually "a:" for the A floppy drive) and under "Data file" put the name you want to use for the file that stores scoring data for this group (here "01-sp" is for my Spring 2001 group – clicking the down arrow brings up other data files); click one or the other to view or print scores from that source. Click "Record" to record scores from a student disk. Students are automatically listed in the box on the top left; click the name of a student to view that student's scores; click "every student" at the bottom to return to normal. To view just some exercises, highlight the ones you are interested in and then click "selected exercises"; to return to viewing all exercises, click "all exercises." To print scores, click "print"; a chart listing scores will be copied into Windows Notepad – and you can print it from there or, if you prefer, copy it into your word processor and print it from there. Depending on the current settings that you see on the screen, your chart will list all the exercises (or just selected ones), from every student (or just one student), from the data file (or from the student disk).

The menu bar at the top lets you do further things. "File" lets you, for example, delete the scores of a student who is no longer in your course. "Options" lets you switch between MultiCola and LogiCola modes,[1] and it lets you set the expected level for LogiCola. "Update" gives an easy way to copy LogiCola scores to student disks. "Timer" is used for timing oral

[1] MultiCola is the program that I wrote to provide computer exercises for my ethics books. The same score processor will record scores from either program. To record LogiCola scores, set it for LogiCola mode using the "Options" menu.

exams (and thus isn't highly relevant to logic. "Help" gives you a help file with further information on the scoring program, lets you update the program from the Web, and tells you the version of the program.

Special codes verify that the scores listed come from the program and not from a student manipulating the score file. If the verification code isn't authentic, the score processor will note that the score is faked and not give the student credit. The code is sophisticated and should be difficult or impossible to break.

Student Disks

I suggest that you give out disks with the program. There are various ways to do this. I buy cheap, already formatted disks in large quantities; I can buy standard 3½ inch floppy disks (1.44 Mb DSDD) for about 20¢ each in quantities of 150 disks. Then I copy the program to the floppies, put a small label (from the drug store) with a student's name on each disk, and give out the disks for free. When disks were more expensive, I used to charge $1 for each disk; you still can do this if you like.

If you don't want to make the disks yourself, you might be able to get your teaching assistant or secretary to do it. Or you might ask your class for a volunteer to make disks for the class (and charge each student maybe $5 for the disk). Or you might have each student download the program from the Web and make his or her own disk.

The score processor under "Update" has handy tools for copying files to student disks. Let's say that you put the LogiCola files (that you got from the compressed file that you downloaded from the Web) in a "C:\LogiCola" folder. Then, under the score processor's "Update" menu, pick "Give update directories" and then type "C:\LogiCola" for the folder with the LogiCola files. Then, to create a LogiCola student disk, pick "Update student disks now w/o recording" from the score processor's "Update" menu. The program files will copy to the floppy disk – and then you'll be asked to insert another disk – and this will continue until the LogiCola program files are copied to all the student disks.[1]

This way of creating student disks actually copies files faster than if you use standard Windows commands for copying files. And it has the advantage that it doesn't copy the NAME.LC and SCORE.LC files; these files will be created to record *your* name and scores if you use LogiCola in that folder – and you don't want them copied to student disks.

I suggest that you make a few extra disks – maybe 5 or 10 extras for a class of 35 students. Disks tend to get lost or go bad; sometimes the metal shutter gets bent, you get disk-read errors, or the program freezes. If such problems occur, give the student a new disk.

Macintosh LogiCola

In late August 2002, I updated the Macintosh version of LogiCola – so now it is identical in essential ways to the Windows and DOS versions. However, the program is new and I am less familiar with the Mac world; so Mac LogiCola may have some bugs (please report to me at gensler@jcu.edu any that you may find). I am unsure if the present version will work on Mac

[1] If you didn't buy already formatted disks, you'll have to format the floppies first.

System 10; if you use System 10, please report to me about if and how LogiCola works on it. If it doesn't work on System 10, I may have to re-do it in a different programming language.

I have not yet created the Mac LogiCola score processor. Until I do, you can use the small Mac COMBINE LC SCORES program to put scores from Mac disks into a single SCORE.LC file – and then you can process this file with the Windows MC-SCORE.EXE score processor. Or you can process Mac LogiCola disks in Windows, as described in the following paragraph.

If you use Windows and never touch a Mac, it's still possible to process scores from Mac LogiCola disks – all from within your Windows-based computer. To do this, you need a program called "Here & Now," made by Software Architects (http://www.softarch.com). This program lets you read Mac disks as if they were Windows/DOS disks – all from your Windows-based computer.[1]

If you've installed "Here & Now," it's very easy to use your Windows-based computer to process scores from Macintosh LogiCola disks. You process them in the same way that you process PC LogiCola disks; just insert the disk and click "Record" on the scoring program.

Computers Without Floppy Drives

If your students have computers without floppy disks, they could copy the SCORE.LC file to a USB flash drive (a little inexpensive device that fits on your key chain, plugs into a USB port of a PC or Mac, and acts like a big floppy disk); then you stick the USB flash drive into the USB port on your computer, change the STUDENT DISK setting in the score processor to "E:" (or whatever it is on your computer – use the BROWSE button if you aren't sure), and then click the RECORD button. Or they could get an external floppy drive that plugs into a USB port; these are readily available, work equally well with PCs and Macs, and could be shared by various students. Or they could e-mail you SCORE.LC as a file attachment; you then open the score processor, go to the directory where your e-mail attachments are stored, and then click the RECORD button. The problem here is that if lots of students do this then your e-mail program may rename the files to keep them from overwriting each other (so you may get SCORE01.LC, SCORE02.LC, and so on) and the score processor won't recognize the renamed files as score files.

Final Remarks on LogiCola

LogiCola can be used for things other than homework. I often use LogiCola in my office, when I work with students individually, as a random-problem generator – for example, to generate an argument that the student will then prove on my blackboard. And I sometimes use LogiCola to generate ideas for what problems to put on a test.

I'll probably make further versions of LogiCola as time go on. Every few months or so, check http://www.harryhiker.com/lc for newer versions.

[1] Here & Now doesn't, however, let you *run* Mac programs on your Windows-based computer; and it seems not to run under Windows XP.

Answers to Problems

This has answers to all the problems in the book, except those for which the book already has the answer.

2.1a

2. This is a wff.
4. This isn't a wff, since "all - is not -" isn't one of our eight forms.
6. This is a wff.
7. This isn't a wff, since a wff that begins with a letter must begin with a small letter.
8. This isn't a wff, since "not all - not -" isn't one of our eight forms.
9. This is a wff.

2.1b

2. t is not s
4. b is G
6. k is g
7. r is B
8. d is b
9. a is S
11. c is m
12. c is L
13. i is G
14. all M is I
16. d is r (where "r" means "the wife of Ralph") or d is R (in a polygamous society, where "R" means "a wife of Ralph")

2.2a

2. This is a syllogism.
4. This is a syllogism.

2.2b

2. some C is B
4. a is C
6. r is not D
7. s is w
8. some C is not P

2.2c

2. x is W   Valid
   x is not Y*
   ∴ some W* is not Y

4. some J is not P*   Valid
   all J* is F
   ∴ some F* is not P

6. all L* is M   Valid
   g is L
   ∴ g* is M*

7. all L* is M   Invalid
   g is not L*
   ∴ g* is not M

8. some N is T   Invalid
   some C is not T*
   ∴ some N* is not C

9. all C* is K   Invalid
   s is K
   ∴ s* is C*

11. s is C   Valid
    s is H
    ∴ some C* is H*

12. some C is H   Invalid
    ∴ some C* is not H

13. a is b   Valid
    b is c
    c is d
    ∴ a* is d*

14. no A* is B*   Invalid
    some B is C
    some D is not C*
    all D* is E
    ∴ some E* is A*

2.3a

2. all C* is F Invalid
 all D* is F
 ∴ all D is C*

4. no U* is P* Invalid
 no F* is U*
 c is F
 ∴ c* is P*

6. no P* is R* Valid
 some P is M
 ∴ some M* is not R

7. all H* is B Invalid
 all C* is B
 ∴ all C is H*

This means "all scrambled eggs are good for breakfast, all coffee with milk is good for breakfast, therefore all coffee with milk is scrambled eggs."

8. b is U Valid
 all U* is O
 ∴ b* is O*

9. b is P Valid
 all P* is J
 ∴ b* is J*

11. all A* is K Valid
 no K* is R*
 ∴ no A is R

12. all M* is R Valid
 all A* is M
 ∴ all A is R*

13. t is P Invalid
 t is L
 all V* is L
 ∴ some V* is P*

14. j is not b* Invalid
 b is L
 ∴ j* is not L

16. all G* is A Valid
 m is not A*
 ∴ m* is not G

17. some M is Q Valid
 no Q* is A*
 ∴ some M* is not A

18. i is H Valid
 i is not D*
 all G* is D
 ∴ some H* is not G

19. all R* is C Valid
 all C* is S
 no F* is S*
 ∴ no F is R

21. all M* is P Valid
 no P* is T*
 ∴ no M is T

22. some B is P Invalid
 some B is T
 ∴ some P* is T*

23. m is B Valid
 m is D
 no D* is A*
 ∴ some B* is not A

24. all T* is O Valid
 r is T
 r is M
 ∴ some M* is O*

2.3b

2. We can't prove either "Carol stole money" or "Carol didn't steal money." Premises 4+9 yield no valid argument with either as the conclusion.
4. David stole money (as we can prove from 5+10+11). And someone besides David stole money (since by 12 the nastiest person stole money and by 6 David is not the nastiest person at the party). So more than one person stole money. We can't prove these using syllogistic logic, but we can using quantificational logic with identity.

2.4a

2. some A is not D
4. all F is D
6. some H is not L
7. all H is R (We could refute this and 8 by finding a poor person who was happy.)
8. all H is R
9. all R is H (We could refute this by finding a rich person who wasn't happy.)
11. no H is S
12. all A is H
13. all S is C
14. g is C (Here "g" = "this group of shirts.")

16. all S is M
17. all M is S
18. all H is L
19. all H is L

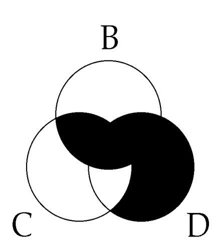
2.5a

2. "Some human acts are not determined."
4. No conclusion validly follows.
6. "Some gospel writers were not apostles."
7. "No cheap waterproof raincoat keeps you dry when hiking uphill" or "Nothing that keeps you dry when hiking uphill is a cheap waterproof raincoat."
8. "All that is or could be experienced is about objects and properties."
9. "No moral judgments are from reason" or "Nothing from reason is a moral judgment."
11. "I am not my mind" or "My mind is not identical to me."
12. "Some acts where you do what you want are not free."
13. "'There is a God' ought to be rejected."
14. "'All unproved beliefs ought to be rejected' ought to be rejected."
16. "Some human beings are not purely selfish."
17. "No virtues are emotions" or "No emotions are virtues."
18. "God is not influenced by anything outside of himself."
19. "God is influenced by everything."
21. "All racial affirmative action programs are wrong."
22. "Some racial affirmative action programs do not discriminate simply because of race."
23. No conclusion validly follows.
24. "Some wrong actions are not blameworthy."

2.6a

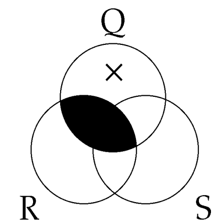
2. Valid

   no B is C
   all D is C
   ∴ no D is B



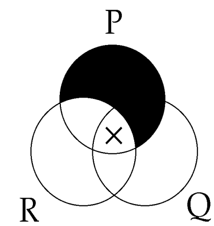
4. Invalid

   no Q is R
   some Q is not S
   ∴ some S is R



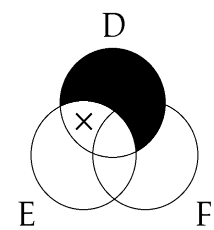
6. Valid

   all P is R
   some Q is P
   ∴ some Q is R



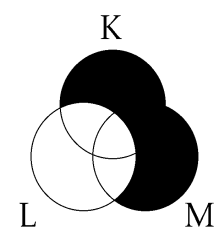
7. Valid

   all D is E
   some D is not F
   ∴ some E is not F



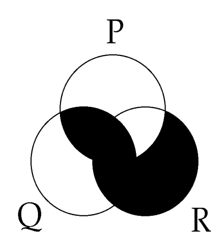
8. Invalid

   all K is L
   all M is L
   ∴ all K is M



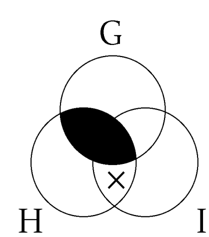
9. Valid

   no P is Q
   all R is P
   ∴ no R is Q

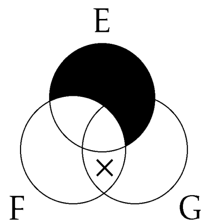


11. Valid

   no G is H
   some H is I
   ∴ some I is not G



12. Invalid
    all E is F
    some G is not E
    ∴ some G is not F



2.7a

2. u is F   Valid
   no S* is F*
   ∴ u* is not S

Premise 2 (implicit) is "No one who studied would have got an F- on the test."

4. all S* is U   Invalid
   some Q is not U*
   ∴ no Q is S

6. i is H   Valid
   i is not D*
   ∴ some H* is not D

7. all P* is N   Valid
   no N* is E*
   ∴ no P is E

8. all W* is S   Valid
   no M* is S*
   ∴ no M is W

Premise 2 (implicit) is "No mathematical knowledge is based on sense experience."

9. all H* is S   Invalid
   some R is not H*
   ∴ some R* is not S

11. j is F   Valid
    j is S
    all S* is W
    ∴ some W* is F*

12. all G* is L   Valid
    no A* is L*
    some A is R
    ∴ some R* is not G

13. all W* is P   Invalid
    all A* is P
    ∴ all W is A*

14. all R* is F   Valid
    all I* is R
    ∴ all I is F*

16. all K* is T   Valid
    all T* is C
    no F* is C*
    ∴ no F is K

Premise 3 (implicit) is "No belief about the future corresponds to the facts."

17. all E* is N   Valid
    no S* is N*
    ∴ no E is S

18. some A is H   Valid
    no A* is S*
    ∴ some H* is not S

19. all I* is M   Invalid
    all C* is M
    ∴ all I is C*

21. no B* is E*   Invalid
    h is E
    h is P
    ∴ no P is B

22. all E* is F   Valid
    no U* is F*
    ∴ no U is E

23. i is T   Invalid
    some T is D
    no D* is K*
    ∴ i* is not K

24. all M* is P   Valid
    all P* is S
    no S* is U*
    ∴ no M is U

26. no T* is F*   Invalid
    some P is F
    ∴ some T* is not P

27. all D* is P   Invalid
    all M* is P
    no S* is M*
    ∴ no S is D

3.1a

2. $(A \cdot (B \vee C))$
4. $(A \supset (B \vee C))$
6. $({\sim}A \supset {\sim}(B \vee C))$
7. $({\sim}A \supset ({\sim}B \vee C))$
8. $((A \vee B) \cdot C)$
9. $(A \vee (B \cdot C))$
11. $(E \supset (B \vee M))$
12. $(C \supset ({\sim}B \supset (T \cdot P)))$
13. $(({\sim}X \cdot M) \supset G)$

14. ~(C ∨ M)
16. (P ⊃ M) [“((M • O) ⊃ W)” is wrong, since the sentence doesn’t mean “If Michigan plays (each other?) and Ohio State plays (each other?) then Michigan will win.” Instead, the sentence means “If Michigan plays Ohio State, then Michigan will win.”]
17. ((D • C) ∨ L)
18. ((H ⊃ J) • F)
19. ((R ∨ F) • L)

3.2a

2.	0	8.	1	13.	1
4.	1	9.	0	14.	0
6.	0	11.	1	16.	1
7.	1	12.	1	17.	0

3.3a

2. (~1 • ~0) = (0 • 1) = 0
4. (1 ⊃ 0) = 0
6. (~1 ⊃ 1) = (0 ⊃ 1) = 1
7. ~(1 ⊃ 0) = ~0 = 1
8. (1 • (0 ∨ 1)) = (1 • 1) = 1
9. (~(0 • 1) ∨ ~1) = (~0 ∨ 0) = (1 ∨ 0) = 1
11. ((1 • ~0) ⊃ ~1) = ((1 • 1) ⊃ 0) = (1 ⊃ 0) = 0
12. ~(1 ⊃ (0 ∨ ~1)) = ~(1 ⊃ (0 ∨ 0)) = ~(1 ⊃ 0) = ~0 = 1
13. (~0 ∨ ~(~1 ≡ 1)) = (1 ∨ ~(0 ≡ 1)) = (1 ∨ ~0) = (1 ∨ 1) = 1
14. (~0 ⊃ (1 • 0)) = (1 ⊃ 0) = 0

3.4a

2. (? ⊃ ~1) = (? ⊃ 0) = ?
4. (~0 • ?) = (1 • ?) = ?
6. (~1 ∨ ?) = (0 ∨ ?) = ?
7. (? ⊃ ~1) = (? ⊃ 0) = ?
8. (~0 ∨ ?) = (1 ∨ ?) = 1
9. (1 • ?) = ?
11. (? • ~1) = (? • 0) = 0
12. (? ∨ 0) = ?

3.5a

2.

P Q	(~P • Q)
0 0	0
0 1	1
1 0	0
1 1	0

4.

P Q R	((P • ~Q) ⊃ R)
0 0 0	1
0 0 1	1
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	1

6.

P Q R	((P ∨ ~Q) ⊃ R)
0 0 0	0
0 0 1	1
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

7.

P Q	(~Q ⊃ ~P)
0 0	1
0 1	1
1 0	0
1 1	1

8.

P	(P ≡ (P • P))
0	1
1	1

9.

P Q R	~(P • (Q ∨ ~R))
0 0 0	1
0 0 1	1
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	0

3.6a

2. Valid: no row has 110.

C D	(C ⊃ D),	~D	∴ ~C
0 0	1	1	1
0 1	1	0	1
1 0	0	1	0
1 1	1	0	0

4. Invalid: rows 2 and 6 have 110.

R L W	((R • L) ⊃ W),	~L	∴ ~W
0 0 0	1	1	1
0 0 1	1	1	0
0 1 0	1	0	1
0 1 1	1	0	0
1 0 0	1	1	1
1 0 1	1	1	0
1 1 0	0	0	1
1 1 1	1	0	0

6. Valid: no row has 1110.

M S G	M,	(M ⊃ S),	(S ⊃ G)	∴ G
0 0 0	0	1	1	0
0 0 1	0	1	1	1
0 1 0	0	1	0	0
0 1 1	0	1	1	1
1 0 0	1	0	1	0
1 0 1	1	0	1	1
1 1 0	1	1	0	0
1 1 1	1	1	1	1

7. Valid: no row has 110.

D G	(D ⊃ G),	~G	∴ ~D
0 0	1	1	1
0 1	1	0	1
1 0	0	1	0
1 1	1	0	0

8. Valid: no row has 1110.

E B R	E,	B,	((E • B) ⊃ R)	∴ R
0 0 0	0	0	1	0
0 0 1	0	0	1	1
0 1 0	0	1	1	0
0 1 1	0	1	1	1
1 0 0	1	0	1	0
1 0 1	1	0	1	1
1 1 0	1	1	0	0
1 1 1	1	1	1	1

9. Valid: no row has 110.

P W	(P ≡ W),	~W	∴ ~P
0 0	1	1	1
0 1	0	0	1
1 0	0	1	0
1 1	1	0	0

11. Valid: no row has 1110.

G C M	(G ⊃ C),	(C ⊃ ~M),	M	∴ ~G
0 0 0	1	1	0	1
0 0 1	1	1	1	1
0 1 0	1	1	0	1
0 1 1	1	0	1	1
1 0 0	0	1	0	0
1 0 1	0	1	1	0
1 1 0	1	1	0	0
1 1 1	1	0	1	0

12. Invalid: row 3 has 1110.

F S O	(F ⊃ (S ∨ O)),	~O,	~F	∴ ~S
0 0 0	1	1	1	1
0 0 1	1	0	1	1
0 1 0	1	1	1	0
0 1 1	1	0	1	0
1 0 0	0	1	0	1
1 0 1	1	0	0	1
1 1 0	1	1	0	0
1 1 1	1	0	0	0

3.7a

2. $((J^1 \cdot {\sim}D^1) \supset Z^0) = 1$ Invalid
$\sim Z^0 = 1$
$D^1 = 1$
$\therefore \sim J^1 = 0$

4. $P^1 = 1$ Invalid
$\therefore (P^1 \cdot Q^0) = 0$

(While we don't initially get a value for Q, we can get true premises and a false conclusion if we make it false.)

6. $((A^1 \cdot U^1) \supset {\sim}B^1) \neq 1$ Valid
$B^1 = 1$
$A^1 = 1$
$\therefore \sim U^1 = 0$

7. $((W^0 \cdot C^1) \supset Z^0) = 1$ Invalid
$\sim Z^0 = 1$
$\therefore \sim C^1 = 0$

(While we don't initially get a value for W, we can get true premises and a false conclusion if we make it false.)

8. $Q^1 = 1$ Valid
$\therefore (P \supset Q^1) \neq 0$

9. $(E^0 \vee (Y \cdot X^0)) \neq 1$ Valid
$\sim E^0 = 1$
$\therefore X^0 = 0$

11. $\sim P^0 = 1$ Invalid

$\therefore$ $\sim(Q^0 \supset P^0) = 0$

(While we don't initially get a value for Q, we can get true premises and a false conclusion if we make it false.)

12. $((\sim M^0 \cdot G^1) \supset R^0) \neq 1$ Valid
    $\sim R^0 = 1$
    $G^1 = 1$
    $\therefore$ $M^0 = 0$
13. $\sim(Q^0 \equiv I^0) \neq 1$ Valid
    $\sim Q^0 = 1$
    $\therefore$ $I^0 = 0$
14. $((Q^1 \cdot R^0) \equiv S^0) = 1$ Invalid
    $Q^1 = 1$
    $\therefore$ $S^0 = 0$

(While we don't initially get a value for R, we can get true premises and a false conclusion if we make it false.)

3.7b

2. $((P \cdot I) \supset O)$ Invalid
   P
   $\sim I$
   $\therefore$ $\sim O$
4. $((P \cdot A) \supset E)$ Valid
   A
   P
   $\therefore$ E
6. $(D \supset (S \cdot R))$ Invalid
   $\sim S$
   $\therefore$ $\sim R$
7. $((M \cdot \sim Y) \supset D)$ Valid
   $\sim D$
   $\sim Y$
   $\therefore$ $\sim M$
8. $((T \cdot \sim U) \supset O)$ Valid
   T
   $\sim O$
   $\therefore$ U
9. $(F \supset (G \equiv O))$ Valid
   F
   O
   $\therefore$ G
11. $(S \supset P)$ Valid
    $\sim P$
    $\therefore$ $\sim S$
12. $((J \cdot \sim V) \supset (R \vee D))$ Invalid
    $\sim R$
    $\sim D$
    $\therefore$ $\sim J$
13. $(I \supset (B \vee N))$ Valid
    $\sim B$
    $\sim N$
    $\therefore$ $\sim I$
14. $(L \supset (C \vee H))$ Invalid
    L
    $\therefore$ H
16. $(K \supset (W \vee P))$ Valid
    $\sim P$
    K
    $\therefore$ W
17. $((M \vee \sim S) \supset V)$ Invalid
    $\sim M$
    S
    $\therefore$ $\sim V$
18. $((M \cdot G) \supset H)$ Valid
    G
    $\sim H$
    $\therefore$ $\sim M$
19. $(I \supset (S \vee C))$ Invalid
    $\sim S$
    $\therefore$ $\sim I$
21. $(M \supset N)$ Valid
    $\sim N$
    $\therefore$ $\sim M$
22. $(M \supset N)$ Valid
    $\sim N$
    $\therefore$ $\sim M$
23. $(R \supset E)$ Valid
    $\sim E$
    $\therefore$ $\sim R$

3.8a

2. $(S \supset G)$ *or, equivalently,* $(\sim G \supset \sim S)$
4. $(T \supset P)$
6. $(M \equiv R)$
7. $(F \vee D)$
8. $(\sim A \cdot \sim H)$ *or, equivalently,* $\sim(A \vee H)$
9. $(N \equiv A)$
11. $(Y \supset I)$
12. $(F \supset \sim K)$
13. $(\sim T \supset \sim K)$
14. $((M \vee F) \cdot \sim(M \cdot F))$

3.9a

2. $(F \supset (W \vee P))$ Valid
   F
   $\sim P$
   $\therefore W$
4. $(M \supset R)$ Invalid
   R
   $\therefore M$
6. $(\sim C \supset \sim S)$ Invalid
   C
   $\therefore S$
7. $((O \cdot \sim N) \supset B)$ Invalid
   N
   $\therefore \sim B$
8. $((D \cdot F) \supset H)$ Valid
   $\sim H$
   D
   $\therefore \sim F$
9. $(F \supset O)$ Invalid
   O
   $\therefore F$
11. $(B \supset T)$ Valid
   B
   $\therefore T$

The implicit premise 2 is "The ball broke the plane of the end zone."

12. $(\sim I \supset (F \vee S))$ Valid
   $\sim S$
   $\sim F$
   $\therefore I$
13. $(C \supset H)$ Invalid
   $\sim C$
   $\therefore \sim H$
14. $(R \supset S)$ Valid
   $\sim S$
   $\therefore \sim R$

The implicit premise 2 is "We don't see the white Appalachian Trail blazes on the trees."

16. $(E \supset A)$ Invalid
   $\sim E$
   $\therefore \sim A$
17. $((T \vee A) \supset B)$ Valid
   T
   $\therefore B$
18. $(R \vee Q)$ Valid
   $\sim R$
   $\therefore Q$

The implicit premise 2 is "You aren't giving me a raise."

19. $(E \supset I)$ Valid
   $\sim I$
   $\therefore \sim E$
21. $(C \supset D)$ Valid
   $\sim D$
   $\therefore \sim C$
22. $(W \supset G)$ Valid
   $\sim G$
   $\therefore \sim W$

The implicit premise 2 is "God doesn't need a cause."

3.10a

2. no conclusion
4. H, I
6. ~Q, B
7. no conclusion
8. no conclusion
9. N, E
11. no conclusion
12. no conclusion
13. P, U
14. ~R, ~S
16. no conclusion
17. ~U, L
18. no conclusion
19. ~Y, ~G

3.11a

2. no conclusion
4. no conclusion
6. no conclusion
7. G
8. Q
9. no conclusion
11. ~N
12. no conclusion
13. no conclusion
14. K
16. L
17. F
18. no conclusion

19. no conclusion

3.12a

2. I
4. no conclusion
6. ~C, ~D
7. no conclusion
8. ~M, I
9. P, ~Q
11. no conclusion
12. ~R
13. ~L, S
14. ~T
16. ~U

3.13a

2. no conclusion
4. ~(I ∨ J)
6. no conclusion
7. ~(A ⊃ B), ~C
8. C
9. no conclusion

4.2a

2. Valid

1 A
[∴ (A ∨ B)
2 asm: ~(A ∨ B)
3 ∴ ~A {from 2}
4 ∴ (A ∨ B) {from 2; 1 contradicts 3}

4. Valid

* 1 ((A ∨ B) ⊃ C)
[∴ (~C ⊃ ~B)
* 2 asm: ~(~C ⊃ ~B)
3 ∴ ~C {from 2}
4 ∴ B {from 2}
* 5 ∴ ~(A ∨ B) {from 1 and 3}
6 ∴ ~A {from 5}
7 ∴ ~B {from 5}
8 ∴ (~C ⊃ ~B) {from 2; 4 contradicts 7}

6. Valid

* 1 (A ⊃ B)
* 2 (B ⊃ C)
[∴ (A ⊃ C)
* 3 asm: ~(A ⊃ C)
4 ∴ A {from 3}
5 ∴ ~C {from 3}
6 ∴ B {from 1 and 4}
7 ∴ ~B {from 2 and 5}
8 ∴ (A ⊃ C) {from 3; 6 contradicts 7}

7. Valid

* 1 (A ≡ B)
[∴ (A ⊃ (A • B))
* 2 asm: ~(A ⊃ (A • B))
* 3 ∴ (A ⊃ B) {from 1}
4 ∴ (B ⊃ A) {from 1}
5 ∴ A {from 2}
* 6 ∴ ~(A • B) {from 2}
7 ∴ B {from 3 and 5}
8 ∴ ~B {from 5 and 6}
9 ∴ (A ⊃ (A • B)) {from 2; 7 contradicts 8}

8. Valid

* 1 ~(A ∨ B)
* 2 (C ∨ B)
* 3 ~(D • C)
[∴ ~D
4 asm: D
5 ∴ ~A {from 1}
6 ∴ ~B {from 1}
7 ∴ C {from 2 and 6}
8 ∴ ~C {from 3 and 4}
9 ∴ ~D {from 4; 7 contradicts 8}

9. Valid

* 1 (A ⊃ B)
2 ~B
[∴ (A ≡ B)
* 3 asm: ~(A ≡ B)
* 4 ∴ (A ∨ B) {from 3}
5 ∴ ~(A • B) {from 3}
6 ∴ ~A {from 1 and 2}
7 ∴ A {from 2 and 4}
8 ∴ (A ≡ B) {from 3; 6 contradicts 7}

4.2b

2. Valid

* 1 (P ⊃ I)
* 2 (I ⊃ ~F)
[∴ (F ⊃ ~P)
* 3 asm: ~(F ⊃ ~P)
4 ∴ F {from 3}
5 ∴ P {from 3}
6 ∴ I {from 1 and 5}

7 ∴ ~I {from 2 and 4}
8 ∴ (F ⊃ ~P) {from 3; 6 contradicts 7}

4. Valid

1 U
2 M
3 W
* 4 ((M • W) ⊃ P)
* 5 ((U • P) ⊃ D)
[∴ D
6 asm: ~D
* 7 ∴ ~(U • P) {from 5 and 6}
8 ∴ ~P {from 1 and 7}
* 9 ∴ ~(M • W) {from 4 and 8}
10 ∴ ~W {from 2 and 9}
11 ∴ D {from 6; 3 contradicts 10}

6. Valid

* 1 (L ⊃ B)
* 2 (L ⊃ ~B)
[∴ ~L
3 asm: L
4 ∴ B {from 1 and 3}
5 ∴ ~B {from 2 and 3}
6 ∴ ~L {from 3; 4 contradicts 5}

7. Valid

1 B
* 2 (B ⊃ C)
* 3 (C ⊃ P)
[∴ P
4 asm: ~P
5 ∴ C {from 1 and 2}
6 ∴ ~C {from 3 and 4}
7 ∴ P {from 4; 5 contradicts 6}

8. Valid

* 1 ((B • ~P) ⊃ C)
* 2 (C ⊃ G)
3 ~G
[∴ (~B ∨ P)
* 4 asm: ~(~B ∨ P)
5 ∴ B {from 4}
6 ∴ ~P {from 4}
7 ∴ ~C {from 2 and 3}
* 8 ∴ ~(B • ~P) {from 1 and 7}
9 ∴ P {from 5 and 8}
10 ∴ (~B ∨ P) {from 4; 6 contradicts 9}

9. Valid

1 G
* 2 ((G • E) ⊃ C)
3 ~C
* 4 (~E ⊃ B)
[∴ B
5 asm: ~B
* 6 ∴ ~(G • E) {from 2 and 3}
7 ∴ E {from 4 and 5}
8 ∴ ~E {from 1 and 6}
9 ∴ B {from 5; 7 contradicts 8}

11. Valid

* 1 (L ⊃ (R ⊃ (A • W)))
2 R
[∴ (L ⊃ W)
* 3 asm: ~(L ⊃ W)
4 ∴ L {from 3}
5 ∴ ~W {from 3}
* 6 ∴ (R ⊃ (A • W)) {from 1 and 4}
* 7 ∴ (A • W) {from 2 and 6}
8 ∴ A {from 7}
9 ∴ W {from 7}
10 ∴ (L ⊃ W) {from 3; 5 contradicts 9}

12. Valid

* 1 (N ⊃ (C • F))
2 O
* 3 ((F • O) ⊃ E)
[∴ (N ⊃ E)
* 4 asm: ~(N ⊃ E)
5 ∴ N {from 4}
6 ∴ ~E {from 4}
* 7 ∴ (C • F) {from 1 and 5}
8 ∴ C {from 7}
9 ∴ F {from 7}
* 10 ∴ ~(F • O) {from 3 and 6}
11 ∴ ~F {from 2 and 10}
12 ∴ (N ⊃ E) {from 4; 9 contradicts 11}

13. Valid

* 1 (I ⊃ (U • ~C))
* 2 (U ⊃ (D ∨ E))
* 3 (D ⊃ A)
4 ~A
* 5 (E ⊃ C)
[∴ ~I
6 asm: I
* 7 ∴ (U • ~C) {from 1 and 6}
8 ∴ U {from 7}
9 ∴ ~C {from 7}
* 10 ∴ (D ∨ E) {from 2 and 8}

11 ∴ ~D {from 3 and 4}
12 ∴ ~E {from 5 and 9}
13 ∴ E {from 10 and 11}
14 ∴ ~I {from 6; 12 contradicts 13}

14. Valid

* 1 (G ⊃ N)
* 2 (N ⊃ (P ∨ F))
* 3 (P ⊃ ~G)
* 4 (N ⊃ ~G)
[∴ ~G
5 asm: G
6 ∴ N {from 1 and 5}
7 ∴ (P ∨ F) {from 2 and 6}
8 ∴ ~P {from 3 and 5}
9 ∴ ~N {from 4 and 5}
10 ∴ ~G {from 5; 6 contradicts 9}

4.3a

2. Invalid

A, ~C, B

* 1 (A ⊃ B)
2 (C ⊃ B)
[∴ (A ⊃ C)
* 3 asm: ~(A ⊃ C)
4 ∴ A {from 3}
5 ∴ ~C {from 3}
6 ∴ B {from 1 and 4}

4. Invalid

~A, B, ~D, C

1 (A ⊃ (B • C))
* 2 (~C ⊃ D)
[∴ ((B • ~D) ⊃ A)
* 3 asm: ~((B • ~D) ⊃ A)
* 4 ∴ (B • ~D) {from 3}
5 ∴ ~A {from 3}
6 ∴ B {from 4}
7 ∴ ~D {from 4}
8 ∴ C {from 2 and 7}

6. Invalid

D, A, ~C, B

* 1 (A ≡ B)
2 (C ⊃ B)
* 3 ~(C • D)
4 D
[∴ ~A
5 asm: A
* 6 ∴ (A ⊃ B) {from 1}
7 ∴ (B ⊃ A) {from 1}
8 ∴ ~C {from 3 and 4}
9 ∴ B {from 5 and 6}

7. Invalid

B, ~C, ~A

* 1 ((A • B) ⊃ C)
[∴ (B ⊃ C)
* 2 asm: ~(B ⊃ C)
3 ∴ B {from 2}
4 ∴ ~C {from 2}
* 5 ∴ ~(A • B) {from 1 and 4}
6 ∴ ~A {from 3 and 5}

8. Invalid

A, E, ~C, ~D, ~B

* 1 ((A • B) ⊃ C)
* 2 ((C ∨ D) ⊃ ~E)
[∴ ~(A • E)
* 3 asm: (A • E)
4 ∴ A {from 3}
5 ∴ E {from 3}
* 6 ∴ ~(C ∨ D) {from 2 and 5}
7 ∴ ~C {from 6}
8 ∴ ~D {from 6}
* 9 ∴ ~(A • B) {from 1 and 7}
10 ∴ ~B {from 4 and 9}

9. Invalid

C, B, ~A

* 1 ~(A • B)
2 (~A ∨ C)
[∴ ~(C • B)
* 3 asm: (C • B)
4 ∴ C {from 3}
5 ∴ B {from 3}
6 ∴ ~A {from 1 and 5}

4.3b

2. Valid

* 1 (V ⊃ (P ∨ A))
* 2 (P ⊃ S)
* 3 (A ⊃ N)
* 4 (~S • ~N)
[∴ ~V
5 asm: V
6 ∴ ~S {from 4}
7 ∴ ~N {from 4}
* 8 ∴ (P ∨ A) {from 1 and 5}
9 ∴ ~P {from 2 and 6}
10 ∴ ~A {from 3 and 7}
11 ∴ A {from 8 and 9}
12 ∴ ~V {from 5; 10 contradicts 11}

4. Invalid

W, M, A

** 1 ((M • ~A) ⊃ I)
2 (I ⊃ W)
[∴ ((M • A) ⊃ ~W)
* 3 asm: ~((M • A) ⊃ ~W)
* 4 ∴ (M • A) {from 3}
5 ∴ W {from 3}
6 ∴ M {from 4}
7 ∴ A {from 4}

6. Invalid

F, ~P, ~B, ~G

* 1 (P ∨ (G ⊃ B))
* 2 (P ⊃ ~F)
* 3 (B ⊃ ~F)
[∴ ~F
4 asm: F
5 ∴ ~P {from 2 and 4}
* 6 ∴ (G ⊃ B) {from 1 and 5}
7 ∴ ~B {from 3 and 4}
8 ∴ ~G {from 6 and 7}

A “G” premise would make it valid.

7. Valid

* 1 (P ⊃ C)
* 2 ((C • D) ⊃ ~G)
[∴ (G ⊃ (~P ∨ ~D))
* 3 ┌ asm: ~(G ⊃ (~P ∨ ~D))
4 │ ∴ G {from 3}
* 5 │ ∴ ~(~P ∨ ~D) {from 3}
6 │ ∴ P {from 5}
7 │ ∴ D {from 5}
8 │ ∴ C {from 1 and 6}
* 9 │ ∴ ~(C • D) {from 2 and 4}
10 └ ∴ ~C {from 7 and 9}
11 ∴ (G ⊃ (~P ∨ ~D)) {from 3; 8 contradicts 10}

8. Invalid

H, ~F, E, ~D

1 (D ⊃ ~F)
* 2 (H ⊃ E)
* 3 (E ⊃ ~D)
[∴ (H ⊃ F)
* 4 asm: ~(H ⊃ F)
5 ∴ H {from 4}
6 ∴ ~F {from 4}
7 ∴ E {from 2 and 5}
8 ∴ ~D {from 3 and 7}

9. Valid

1 G
2 ~B
3 O
* 4 ((G • ~B) ⊃ C)
* 5 ((C • O) ⊃ R)
[∴ R
6 ┌ asm: ~R
* 7 │ ∴ ~(C • O) {from 5 and 6}
8 │ ∴ ~C {from 3 and 7}
* 9 │ ∴ ~(G • ~B) {from 4 and 8}
10 └ ∴ B {from 1 and 9}
11 ∴ R {from 6; 2 contradicts 10}

11. Invalid

T, ~I, E, ~M

1 (M ⊃ ~I)
2 (I ⊃ ~M)
* 3 (E ⊃ ~M)
* 4 (T ⊃ E)
[∴ (T ⊃ I)
* 5 asm: ~(T ⊃ I)
6 ∴ T {from 5}
7 ∴ ~I {from 5}
8 ∴ E {from 4 and 6}
9 ∴ ~M {from 3 and 8}

12. Valid

* 1 ((D • C) ⊃ U)
* 2 (U ⊃ O)
* 3 ((D • R) ⊃ U)
[∴ (D ⊃ (O ∨ (~C • ~R)))
* 4 ┌ asm: ~(D ⊃ (O ∨ (~C • ~R)))
5 │ ∴ D {from 4}
* 6 │ ∴ ~(O ∨ (~C • ~R)) {from 4}
7 │ ∴ ~O {from 6}
* 8 │ ∴ ~(~C • ~R) {from 6}
9 │ ∴ ~U {from 2 and 7}
*10 │ ∴ ~(D • C) {from 1 and 9}
*11 │ ∴ ~(D • R) {from 3 and 9}
12 │ ∴ ~C {from 5 and 10}
13 │ ∴ R {from 8 and 12}
14 └ ∴ ~R {from 5 and 11}
15 ∴ (D ⊃ (O ∨ (~C • ~R))) {from 4; 13 contradicts 14}

13. Invalid

~W, ~P, V, D

* 1 (P ⊃ W)
* 2 ((~D • V) ⊃ W)
3 (P ⊃ ~D)
[∴ ((P ∨ V) ⊃ W)
* 4 asm: ~((P ∨ V) ⊃ W)
* 5 ∴ (P ∨ V) {from 4}

6 ∴ ~W {from 4}
7 ∴ ~P {from 1 and 6}
* 8 ∴ ~(~D • V) {from 2 and 6}
9 ∴ V {from 5 and 7}
10 ∴ D {from 8 and 9}

14. Valid

* 1 ((C • N) ⊃ Y)
2 ~Y
[∴ (C ⊃ ~N)
* 3 asm: ~(C ⊃ ~N)
4 ∴ C {from 3}
5 ∴ N {from 3}
* 6 ∴ ~(C • N) {from 1 and 2}
7 ∴ ~N {from 4 and 6}
8 ∴ (C ⊃ ~N) {from 3; 5 contradicts 7}

16. Valid

1 A
* 2 (A ⊃ (C • ~N))
* 3 (~N ⊃ ~K)
[∴ ~K
4 asm: K
* 5 ∴ (C • ~N) {from 1 and 2}
6 ∴ C {from 5}
7 ∴ ~N {from 5}
8 ∴ N {from 3 and 4}
9 ∴ ~K {from 4; 7 contradicts 8}

17. Valid

* 1 (C ⊃ (W ∨ B))
* 2 (B ⊃ ~P)
* 3 (P ⊃ ~W)
[∴ (P ⊃ ~C)
* 4 asm: ~(P ⊃ ~C)
5 ∴ P {from 4}
6 ∴ C {from 4}
* 7 ∴ (W ∨ B) {from 1 and 6}
8 ∴ ~B {from 2 and 5}
9 ∴ ~W {from 3 and 5}
10 ∴ W {from 7 and 8}
11 ∴ (P ⊃ ~C) {from 4; 9 contradicts 10}

18. Invalid

~D, R, ~B

1 (B ⊃ R)
2 ~D
* 3 (B ⊃ D)
[∴ ~R
4 asm: R
5 ∴ ~B {from 2 and 3}

19. Valid

1 E
* 2 (~R ⊃ F)
* 3 ((E • F) ⊃ H)
[∴ (R ∨ H)
* 4 asm: ~(R ∨ H)
5 ∴ ~R {from 4}
6 ∴ ~H {from 4}
7 ∴ F {from 2 and 5}
* 8 ∴ ~(E • F) {from 3 and 6}
9 ∴ ~F {from 1 and 8}
10 ∴ (R ∨ H) {from 4; 7 contradicts 9}

21. Valid

* 1 ((R • ~K) ⊃ O)
* 2 ((~R • ~K) ⊃ O)
3 ~K
[∴ O
4 asm: ~O
* 5 ∴ ~(R • ~K) {from 1 and 4}
* 6 ∴ ~(~R • ~K) {from 2 and 4}
7 ∴ ~R {from 3 and 5}
8 ∴ R {from 3 and 6}
9 ∴ O {from 4; 7 contradicts 8}

22. Invalid

~E, ~S, ~K

1 (K ⊃ ~E)
* 2 (K ⊃ S)
* 3 (S ⊃ E)
[∴ E
4 asm: ~E
5 ∴ ~S {from 3 and 4}
6 ∴ ~K {from 2 and 5}

23. Invalid

I, ~G, ~P, W

1 I
* 2 (I ⊃ (W ∨ P))
* 3 (P ⊃ G)
[∴ G
4 asm: ~G
* 5 ∴ (W ∨ P) {from 1 and 2}
6 ∴ ~P {from 3 and 4}
7 ∴ W {from 5 and 6}

24. Valid

1 D
2 O
* 3 (O ⊃ ~C)
* 4 ((D • ~C) ⊃ ~M)
* 5 (R ⊃ M)

[∴ ~R
6 asm: R
7 ∴ ~C {from 2 and 3}
8 ∴ M {from 5 and 6}
* 9 ∴ ~(D • ~C) {from 4 and 8}
10 ∴ C {from 1 and 9}
11 ∴ ~R {from 6; 7 contradicts 10}

4.5a

2. Valid

* 1 (((A • B) ⊃ C) ⊃ (D ⊃ E))
2 D
[∴ (C ⊃ E)
* 3 asm: ~(C ⊃ E)
4 ∴ C {from 3}
5 ∴ ~E {from 3}
6 asm: ~((A • B) ⊃ C) {break up 1}
7 ∴ (A • B) {from 6}
8 ∴ ~C {from 6}
9 ∴ ((A • B) ⊃ C) {from 6; 4 contradicts 8}
* 10 ∴ (D ⊃ E) {from 1 and 9}
11 ∴ E {from 2 and 10}
12 ∴ (C ⊃ E) {from 3; 5 contradicts 11}

4. Valid

* 1 (A ∨ (D • E))
2 (A ⊃ (B • C))
[∴ (D ∨ C)
* 3 asm: ~(D ∨ C)
4 ∴ ~D {from 3}
5 ∴ ~C {from 3}
6 asm: A {break up 1}
7 ∴ (B • C) {from 2 and 6}
8 ∴ B {from 7}
9 ∴ C {from 7}
10 ∴ ~A {from 6; 5 contradicts 9}
11 ∴ (D • E) {from 1 and 10}
12 ∴ D {from 11}
13 ∴ (D ∨ C) {from 3; 4 contradicts 12}

6. Valid

* 1 (~(A ∨ B) ⊃ (C ⊃ D))
* 2 (~A • ~D)
[∴ (~B ⊃ ~C)
* 3 asm: ~(~B ⊃ ~C)
4 ∴ ~A {from 2}
5 ∴ ~D {from 2}
6 ∴ ~B {from 3}
7 ∴ C {from 3}
8 asm: (A ∨ B) {break up 1}
9 ∴ B {from 4 and 8}
10 ∴ ~(A ∨ B) {from 8; 6 contradicts 9}
* 11 ∴ (C ⊃ D) {from 1 and 10}
12 ∴ ~C {from 5 and 11}
13 ∴ (~B ⊃ ~C) {from 3; 7 contradicts 12}

7. Valid

* 1 (~A ≡ B)
[∴ ~(A ≡ B)
* 2 asm: (A ≡ B)
* 3 ∴ (~A ⊃ B) {from 1}
4 ∴ (B ⊃ ~A) {from 1}
5 ∴ (A ⊃ B) {from 2}
* 6 ∴ (B ⊃ A) {from 2}
7 asm: A {break up 3}
8 ∴ ~B {from 4 and 7}
9 ∴ B {from 5 and 7}
10 ∴ ~A {from 7; 8 contradicts 9}
11 ∴ B {from 3 and 10}
12 ∴ ~B {from 6 and 10}
13 ∴ ~(A ≡ B) {from 2; 11 contradicts 12}

8. Valid

* 1 (A ⊃ (B • ~C))
2 C
3 ((D • ~E) ∨ A)
[∴ D
4 asm: ~D
5 asm: ~A {break up 1}
6 ∴ (D • ~E) {from 3 and 5}
7 ∴ D {from 6}
8 ∴ A {from 5; 4 contradicts 7}
* 9 ∴ (B • ~C) {from 1 and 8}
10 ∴ B {from 9}
11 ∴ ~C {from 9}
12 ∴ D {from 4; 2 contradicts 11}

4.5b

2. Valid

* 1 ((C • E) ⊃ (W • A))
2 (~E ⊃ (D • A))
[∴ (C ⊃ A)
* 3 asm: ~(C ⊃ A)
4 ∴ C {from 3}
5 ∴ ~A {from 3}
6 asm: ~(C • E) {break up 1}
7 ∴ ~E {from 4 and 6}
8 ∴ (D • A) {from 2 and 7}

9 ∴ D {from 8}
10 ∴ A {from 8}
*11 ∴ (C • E) {from 6; 5 contradicts 10}
12 ∴ E {from 11}
*13 ∴ (W • A) {from 1 and 11}
14 ∴ W {from 13}
15 ∴ A {from 13}
16 ∴ (C ⊃ A) {from 3; 5 contradicts 15}

4. Valid

* 1 (K ⊃ (L • R))
2 (~K ⊃ (I • R))
[∴ R
3 asm: ~R
4 asm: ~K {break up 1}
5 ∴ (I • R) {from 2 and 4}
6 ∴ I {from 5}
7 ∴ R {from 5}
8 ∴ K {from 4; 3 contradicts 7}
* 9 ∴ (L • R) {from 1 and 8}
10 ∴ L {from 9}
11 ∴ R {from 9}
12 ∴ R {from 3; 3 contradicts 11}

6. Valid

* 1 (T ∨ F)
2 (T ⊃ (S ∨ O))
3 ~S
4 ~O
[∴ (F • ~T)
* 5 asm: ~(F • ~T)
6 asm: T {break up 1}
7 ∴ (S ∨ O) {from 2 and 6}
8 ∴ O {from 3 and 7}
9 ∴ ~T {from 6; 4 contradicts 8}
10 ∴ F {from 1 and 9}
11 ∴ ~F {from 5 and 9}
12 ∴ (F • ~T) {from 5; 10 contradicts 11}

7. Valid

* 1 ((W ∨ T) ⊃ (R • H))
[∴ (~H ⊃ ~T)
* 2 asm: ~(~H ⊃ ~T)
3 ∴ ~H {from 2}
4 ∴ T {from 2}
5 asm: ~(W ∨ T) {break up 1}
6 ∴ ~W {from 5}
7 ∴ ~T {from 5}
8 ∴ (W ∨ T) {from 5; 4 contradicts 7}
* 9 ∴ (R • H) {from 1 and 8}
10 ∴ R {from 9}
11 ∴ H {from 9}
12 ∴ (~H ⊃ ~T) {from 2; 3 contradicts 11}

8. Valid

1 M
* 2 ((M • E) ⊃ (B ∨ G))
3 ~G
[∴ (~E ∨ B)
* 4 asm: ~(~E ∨ B)
5 ∴ E {from 4}
6 ∴ ~B {from 4}
7 asm: ~(M • E) {break up 2}
8 ∴ ~E {from 1 and 7}
9 ∴ (M • E) {from 7; 5 contradicts 8}
*10 ∴ (B ∨ G) {from 2 and 9}
11 ∴ B {from 3 and 10}
12 ∴ (~E ∨ B) {from 4; 6 contradicts 11}

9. Valid

* 1 ((D • P) ⊃ (T ⊃ E))
* 2 ((T • E) ⊃ ~P)
[∴ (D ⊃ (~P ∨ ~T))
* 3 asm: ~(D ⊃ (~P ∨ ~T))
4 ∴ D {from 3}
* 5 ∴ ~(~P ∨ ~T) {from 3}
6 ∴ P {from 5}
7 ∴ T {from 5}
* 8 ∴ ~(T • E) {from 2 and 6}
9 ∴ ~E {from 7 and 8}
10 asm: ~(D • P) {break up 1}
11 ∴ ~P {from 4 and 10}
12 ∴ (D • P) {from 10; 6 contradicts 11}
*13 ∴ (T ⊃ E) {from 1 and 12}
14 ∴ E {from 7 and 13}
15 ∴ (D ⊃ (~P ∨ ~T)) {from 3; 9 contradicts 14}

4.6a

2. Invalid

$\boxed{\text{A, B}}$

1 (~A ⊃ B)
[∴ ~(A ⊃ B)
**2 asm: (A ⊃ B)
3 asm: A {break up 1}
4 ∴ B {from 2 and 3}

4. Invalid

$\boxed{\text{~A, ~B}}$

1 ~(A • B)
[∴ ~(A ≡ B)

* 2 asm: (A ≡ B)
3 ∴ (A ⊃ B) {from 2}
** 4 ∴ (B ⊃ A) {from 2}
5 asm: ~A {break up 1}
6 ∴ ~B {from 4 and 5}

6. Invalid

~A, B

1 (~A ∨ ~B)
[∴ ~(A ∨ B)
** 2 asm: (A ∨ B)
3 asm: ~A {break up 1}
4 ∴ B {from 2 and 3}

7. Invalid

~E, ~C, ~A, D

1 (A ⊃ (B • C))
** 2 ((D ⊃ E) ⊃ A)
[∴ (E ∨ C)
* 3 asm: ~(E ∨ C)
4 ∴ ~E {from 3}
5 ∴ ~C {from 3}
6 asm: ~A {break up 1}
** 7 ∴ ~(D ⊃ E) {from 2 and 6}
8 ∴ D {from 7}

8. Invalid

D, ~A, B

1 (A ⊃ (B ⊃ C))
2 (B ∨ ~(C ⊃ D))
[∴ (D ⊃ ~(A ∨ B))
* 3 asm: ~(D ⊃ ~(A ∨ B))
4 ∴ D {from 3}
** 5 ∴ (A ∨ B) {from 3}
6 asm: ~A {break up 1}
7 ∴ B {from 5 and 6}

4.6b

2. Valid

* 1 (L ⊃ (I • T))
2 (~L ⊃ (D • T))
[∴ T
3 asm: ~T
4 asm: ~L {break up 1}
5 ∴ (D • T) {from 2 and 4}
6 ∴ D {from 5}
7 ∴ T {from 5}
8 ∴ L {from 4; 3 contradicts 7}
* 9 ∴ (I • T) {from 1 and 8}
10 ∴ I {from 9}
11 ∴ T {from 9}
12 ∴ T {from 3; 3 contradicts 11}

4. Valid

* 1 (~A ⊃ (L ∨ C))
2 ((~L • ~C) ⊃ O)
3 ~L
[∴ (~C ⊃ (A • O))
* 4 asm: ~(~C ⊃ (A • O))
5 ∴ ~C {from 4}
6 ∴ ~(A • O) {from 4}
7 asm: A {break up 1}
8 ∴ ~O {from 6 and 7}
9 ∴ ~(~L • ~C) {from 2 and 8}
10 ∴ C {from 3 and 9}
11 ∴ ~A {from 7; 5 contradicts 10}
* 12 ∴ (L ∨ C) {from 1 and 11}
13 ∴ C {from 3 and 12}
14 ∴ (~C ⊃ (A • O)) {from 4; 5 contradicts 13}

6. Valid

* 1 ((T • G) ⊃ (E ∨ D))
2 ~E
3 ~D
* 4 (~G ⊃ ~T)
[∴ ~T
5 asm: T
6 ∴ G {from 4 and 5}
7 asm: ~(T • G) {break up 1}
8 ∴ ~G {from 5 and 7}
9 ∴ (T • G) {from 7; 6 contradicts 8}
* 10 ∴ (E ∨ D) {from 1 and 9}
11 ∴ D {from 2 and 10}
12 ∴ ~T {from 5; 3 contradicts 11}

7. Valid

* 1 (T ⊃ (W ∨ P))
* 2 (W ⊃ (E • F))
3 (P ⊃ (L • F))
[∴ (T ⊃ F)
* 4 asm: ~(T ⊃ F)
5 ∴ T {from 4}
6 ∴ ~F {from 4}
7 ∴ (W ∨ P) {from 1 and 5}
8 asm: ~W {break up 2}
9 ∴ P {from 7 and 8}
10 ∴ (L • F) {from 3 and 9}
11 ∴ L {from 10}
12 ∴ F {from 10}
13 ∴ W {from 8; 6 contradicts 12}
* 14 ∴ (E • F) {from 2 and 13}
15 ∴ E {from 14}

16 ∴ F {from 14}
17 ∴ (T ⊃ F) {from 4; 6 contradicts 16}

8. Invalid

~C, ~M, ~S

1 (M ⊃ (F • S))
2 (S ⊃ (W • C))
[∴ (~M ⊃ C)
* 3 asm: ~(~M ⊃ C)
4 ∴ ~M {from 3}
5 ∴ ~C {from 3}
6 asm: ~S {break up 2}

9. Valid

* 1 (F ≡ (H • L))
* 2 (A ⊃ H)
3 A
[∴ (F ≡ L)
* 4 asm: ~(F ≡ L)
* 5 ∴ (F ⊃ (H • L)) {from 1}
6 ∴ ((H • L) ⊃ F) {from 1}
7 ∴ (F ∨ L) {from 4}
* 8 ∴ ~(F • L) {from 4}
9 ∴ H {from 2 and 3}
10 asm: ~F {break up 5}
11 ∴ ~(H • L) {from 6 and 10}
12 ∴ L {from 7 and 10}
13 ∴ ~L {from 9 and 11}
14 ∴ F {from 10; 12 contradicts 13}
*15 ∴ (H • L) {from 5 and 14}
16 ∴ L {from 15}
17 ∴ ~L {from 8 and 14}
18 ∴ (F ≡ L) {from 4; 16 contradicts 17}

11. Invalid

H, ~C, ~A

1 (A ⊃ (H • L))
2 (C ⊃ A)
[∴ (~C ⊃ ~H)
* 3 asm: ~(~C ⊃ ~H)
4 ∴ ~C {from 3}
5 ∴ H {from 3}
6 asm: ~A {break up 1}

12. Valid

* 1 (K ⊃ (E ∨ L))
* 2 (~M ⊃ (~E • ~L))
3 (M ⊃ (S • F))
4 ~F
[∴ ~K
5 asm: K
* 6 ∴ (E ∨ L) {from 1 and 5}
7 asm: M {break up 2}
8 ∴ (S • F) {from 3 and 7}
9 ∴ S {from 8}
10 ∴ F {from 8}
11 ∴ ~M {from 7; 4 contradicts 10}
*12 ∴ (~E • ~L) {from 2 and 11}
13 ∴ ~E {from 12}
14 ∴ ~L {from 12}
15 ∴ L {from 6 and 13}
16 ∴ ~K {from 5; 14 contradicts 15}

13. Valid

* 1 (P ⊃ (D ∨ V))
* 2 (D ⊃ ~M)
* 3 ((V • M) ⊃ Q)
4 ~Q
* 5 ((P • ~M) ⊃ S)
[∴ (P ⊃ (S • ~M))
* 6 asm: ~(P ⊃ (S • ~M))
7 ∴ P {from 6}
* 8 ∴ ~(S • ~M) {from 6}
9 ∴ (D ∨ V) {from 1 and 7}
10 ∴ ~(V • M) {from 3 and 4}
11 asm: ~D {break up 2}
12 ∴ V {from 9 and 11}
13 ∴ ~M {from 10 and 12}
14 ∴ ~S {from 8 and 13}
15 ∴ ~(P • ~M) {from 5 and 14}
16 ∴ M {from 7 and 15}
17 ∴ D {from 11; 13 contradicts 16}
18 ∴ ~M {from 2 and 17}
19 ∴ ~S {from 8 and 18}
*20 ∴ ~(P • ~M) {from 5 and 19}
21 ∴ ~P {from 18 and 20}
22 ∴ (P ⊃ (S • ~M)) {from 6; 7 contradicts 21}

14. Valid

* 1 ((R • I) ⊃ (F • M))
* 2 (I ⊃ R)
[∴ (I ⊃ M)
* 3 asm: ~(I ⊃ M)
4 ∴ I {from 3}
5 ∴ ~M {from 3}
6 ∴ R {from 2 and 4}
7 asm: ~(R • I) {break up 1}
8 ∴ ~R {from 4 and 7}
9 ∴ (R • I) {from 7; 6 contradicts 8}
*10 ∴ (F • M) {from 1 and 9}
11 ∴ F {from 10}
12 ∴ M {from 10}
13 ∴ (I ⊃ M) {from 3; 5 contradicts 12}

16. Invalid

B, ~L, S, ~T

1 ((B • ~S) ⊃ ~K)
2 ((T • ~K) ⊃ (M • L))
[∴ (B ⊃ L)
* 3 asm: ~(B ⊃ L)
4 ∴ B {from 3}
5 ∴ ~L {from 3}
** 6 asm: ~(B • ~S) {break up 1}
7 ∴ S {from 4 and 6}
8 asm: ~(T • ~K) {break up 2}
9 asm: ~T {break up 8}

5.1a

2. (Cx ∨ Ex)
4. (∃x)~Ex
6. (Lx ⊃ Ex)
7. (x)(Lx ⊃ Ex)
8. ~(∃x)Ex
9. (∃x)(Lx • Ex)
11. (∃x)(Lx • (Ex • Cx))
12. ~(∃x)((Lx • Rx) • Ex)
13. ~(x)Rx
14. (x)(Lx ⊃ (Cx ∨ Ex))
16. (x)((Cx • Lx) ⊃ Ex)
17. ~(x)(~Lx ⊃ Ex)
18. (∃x)((Lx • ~Cx) • Rx)
19. ~(∃x)((Cx ∨ Ex) • Rx)
21. ~(x)(Cx ∨ Ex)
22. ~(x)((Ex ∨ Cx) ⊃ Lx)
23. (x)((Ex ∨ Cx) ⊃ Lx) *or, equivalently,* ((x)(Ex ⊃ Lx) • (x)(Cx ⊃ Lx)). [The more obvious "(x)((Ex • Cx) ⊃ Lx)" is wrong because it says that all who are evil *and* crazy are logicians – which isn't what the English sentence means.]
24. (x)(Ex • Lx)

5.2a

2. Valid

* 1 ~(∃x)(Fx • ~Gx)
[∴ (x)(Fx ⊃ Gx)
* 2 asm: ~(x)(Fx ⊃ Gx)
3 ∴ (x)~(Fx • ~Gx) {from 1}
* 4 ∴ (∃x)~(Fx ⊃ Gx) {from 2}
* 5 ∴ ~(Fa ⊃ Ga) {from 4}
6 ∴ Fa {from 5}
7 ∴ ~Ga {from 5}
* 8 ∴ ~(Fa • ~Ga) {from 3}
9 ∴ Ga {from 6 and 8}
10 ∴ (x)(Fx ⊃ Gx) {from 2; 7 contradicts 9}

4. Valid

1 (x)((Fx ∨ Gx) ⊃ Hx)
[∴ (x)(~Hx ⊃ ~Fx)
* 2 asm: ~(x)(~Hx ⊃ ~Fx)
* 3 ∴ (∃x)~(~Hx ⊃ ~Fx) {from 2}
* 4 ∴ ~(~Ha ⊃ ~Fa) {from 3}
5 ∴ ~Ha {from 4}
6 ∴ Fa {from 4}
* 7 ∴ ((Fa ∨ Ga) ⊃ Ha) {from 1}
8 ∴ ~(Fa ∨ Ga) {from 5 and 7}
9 ∴ ~Fa {from 8}
10 ∴ (x)(~Hx ⊃ ~Fx) {from 2; 6 contradicts 9}

6. Valid

1 (x)(Fx ∨ Gx)
* 2 ~(x)Fx
[∴ (∃x)Gx
* 3 asm: ~(∃x)Gx
* 4 ∴ (∃x)~Fx {from 2}
5 ∴ (x)~Gx {from 3}
6 ∴ ~Fa {from 4}
* 7 ∴ (Fa ∨ Ga) {from 1}
8 ∴ Ga {from 6 and 7}
9 ∴ ~Ga {from 5}
10 ∴ (∃x)Gx {from 3; 8 contradicts 9}

7. Valid

1 (x)~(Fx ∨ Gx)
[∴ (x)~Fx
* 2 asm: ~(x)~Fx
* 3 ∴ (∃x)Fx {from 2}
4 ∴ Fa {from 3}
5 ∴ ~(Fa ∨ Ga) {from 1}
6 ∴ ~Fa {from 5}
7 ∴ (x)~Fx {from 2; 4 contradicts 6}

8. Valid

1 (x)(Fx ⊃ Gx)
2 (x)(Fx ⊃ ~Gx)
[∴ (x)~Fx
* 3 asm: ~(x)~Fx
* 4 ∴ (∃x)Fx {from 3}
5 ∴ Fa {from 4}
* 6 ∴ (Fa ⊃ Ga) {from 1}
7 ∴ Ga {from 5 and 6}
* 8 ∴ (Fa ⊃ ~Ga) {from 2}
9 ∴ ~Ga {from 5 and 8}

10 ∴ (x)~Fx {from 3; 7 contradicts 9}

9. Valid

1 (x)(Fx ⊃ Gx)
2 (x)(~Fx ⊃ Hx)
[∴ (x)(Gx ∨ Hx)
* 3 asm: ~(x)(Gx ∨ Hx)
* 4 ∴ (∃x)~(Gx ∨ Hx) {from 3}
* 5 ∴ ~(Ga ∨ Ha) {from 4}
6 ∴ ~Ga {from 5}
7 ∴ ~Ha {from 5}
* 8 ∴ (Fa ⊃ Ga) {from 1}
9 ∴ ~Fa {from 6 and 8}
* 10 ∴ (~Fa ⊃ Ha) {from 2}
11 ∴ Fa {from 7 and 10}
12 ∴ (x)(Gx ∨ Hx) {from 3; 9 contradicts 11}

5.2b

2. Valid

1 (x)Mx
[∴ (x)(Lx ⊃ Mx)
* 2 asm: ~(x)(Lx ⊃ Mx)
* 3 ∴ (∃x)~(Lx ⊃ Mx) {from 2}
* 4 ∴ ~(La ⊃ Ma) {from 3}
5 ∴ La {from 4}
6 ∴ ~Ma {from 4}
7 ∴ Ma {from 1}
8 ∴ (x)(Lx ⊃ Mx) {from 2; 6 contradicts 7}

4. Valid

1 (x)(Jx ⊃ Ux)
2 (x)(~Jx ⊃ Dx)
[∴ (x)(Ux ∨ Dx)
* 3 asm: ~(x)(Ux ∨ Dx)
* 4 ∴ (∃x)~(Ux ∨ Dx) {from 3}
* 5 ∴ ~(Ua ∨ Da) {from 4}
6 ∴ ~Ua {from 5}
7 ∴ ~Da {from 5}
* 8 ∴ (Ja ⊃ Ua) {from 1}
9 ∴ ~Ja {from 6 and 8}
* 10 ∴ (~Ja ⊃ Da) {from 2}
11 ∴ Ja {from 7 and 10}
12 ∴ (x)(Ux ∨ Dx) {from 3; 9 contradicts 11}

6. Valid

* 1 ~(∃x)(Px • Bx)
* 2 (∃x)(Cx • Bx)
[∴ (∃x)(Cx • ~Px)
* 3 asm: ~(∃x)(Cx • ~Px)
4 ∴ (x)~(Px • Bx) {from 1}
* 5 ∴ (Ca • Ba) {from 2}
6 ∴ (x)~(Cx • ~Px) {from 3}
7 ∴ Ca {from 5}
8 ∴ Ba {from 5}
* 9 ∴ ~(Pa • Ba) {from 4}
10 ∴ ~Pa {from 8 and 9}
* 11 ∴ ~(Ca • ~Pa) {from 6}
12 ∴ Pa {from 7 and 11}
13 ∴ (∃x)(Cx • ~Px) {from 3; 10 contradicts 12}

7. Valid

1 (x)(~Wx ⊃ Ax)
[∴ (x)(~Ax ⊃ Wx)
* 2 asm: ~(x)(~Ax ⊃ Wx)
* 3 ∴ (∃x)~(~Ax ⊃ Wx) {from 2}
* 4 ∴ ~(~Aa ⊃ Wa) {from 3}
5 ∴ ~Aa {from 4}
6 ∴ ~Wa {from 4}
* 7 ∴ (~Wa ⊃ Aa) {from 1}
8 ∴ Wa {from 5 and 7}
9 ∴ (x)(~Ax ⊃ Wx) {from 2; 6 contradicts 8}

8. Valid

1 (x)(Bx ⊃ Dx)
[∴ (x)((Bx • Mx) ⊃ Dx)
* 2 asm: ~(x)((Bx • Mx) ⊃ Dx)
* 3 ∴ (∃x)~((Bx • Mx) ⊃ Dx) {from 2}
* 4 ∴ ~((Ba • Ma) ⊃ Da) {from 3}
* 5 ∴ (Ba • Ma) {from 4}
6 ∴ ~Da {from 4}
7 ∴ Ba {from 5}
8 ∴ Ma {from 5}
* 9 ∴ (Ba ⊃ Da) {from 1}
10 ∴ ~Ba {from 6 and 9}
11 ∴ (x)((Bx • Mx) ⊃ Dx) {from 2; 7 contradicts 10}

9. Valid

* 1 (∃x)((Lx • Ux) • ~Wx)
[∴ ~(x)(Lx ⊃ Wx)
2 asm: (x)(Lx ⊃ Wx)
* 3 ∴ ((La • Ua) • ~Wa) {from 1}
* 4 ∴ (La • Ua) {from 3}
5 ∴ ~Wa {from 3}
6 ∴ La {from 4}
7 ∴ Ua {from 4}
* 8 ∴ (La ⊃ Wa) {from 2}

9 ∴ ~La {from 5 and 8}
10 ∴ ~(x)(Lx ⊃ Wx) {from 2; 6 contradicts 9}

11. Valid

* 1 ~(∃x)(Tx • Mx)
2 (x)(Cx ⊃ Mx)
3 (x)(Cx ⊃ Tx)
[∴ ~(∃x)Cx
* 4 asm: (∃x)Cx
5 ∴ (x)~(Tx • Mx) {from 1}
6 ∴ Ca {from 4}
* 7 ∴ (Ca ⊃ Ma) {from 2}
8 ∴ Ma {from 6 and 7}
* 9 ∴ (Ca ⊃ Ta) {from 3}
10 ∴ Ta {from 6 and 9}
* 11 ∴ ~(Ta • Ma) {from 5}
12 ∴ ~Ta {from 8 and 11}
13 ∴ ~(∃x)Cx {from 4; 10 contradicts 12}

12. Valid

1 (x)(Gx ⊃ (Ex ∨ Dx))
* 2 ~(∃x)(Mx • Ex)
* 3 ~(∃x)(Mx • Dx)
[∴ ~(∃x)(Mx • Gx)
* 4 asm: (∃x)(Mx • Gx)
5 ∴ (x)~(Mx • Ex) {from 2}
6 ∴ (x)~(Mx • Dx) {from 3}
* 7 ∴ (Ma • Ga) {from 4}
8 ∴ Ma {from 7}
9 ∴ Ga {from 7}
* 10 ∴ (Ga ⊃ (Ea ∨ Da)) {from 1}
* 11 ∴ (Ea ∨ Da) {from 9 and 10}
* 12 ∴ ~(Ma • Ea) {from 5}
13 ∴ ~Ea {from 8 and 12}
14 ∴ Da {from 11 and 13}
* 15 ∴ ~(Ma • Da) {from 6}
16 ∴ ~Da {from 8 and 15}
17 ∴ ~(∃x)(Mx • Gx) {from 4; 14 contradicts 16}

13. Valid

1 (x)(Tx ⊃ Wx)
2 (x)(Wx ⊃ Ox)
3 (x)(~Tx ⊃ Ox)
[∴ (x)Ox
* 4 asm: ~(x)Ox
* 5 ∴ (∃x)~Ox {from 4}
6 ∴ ~Oa {from 5}
* 7 ∴ (Ta ⊃ Wa) {from 1}
* 8 ∴ (Wa ⊃ Oa) {from 2}
9 ∴ ~Wa {from 6 and 8}
10 ∴ ~Ta {from 7 and 9}
* 11 ∴ (~Ta ⊃ Oa) {from 3}
12 ∴ Ta {from 6 and 11}
13 ∴ (x)Ox {from 4; 10 contradicts 12}

5.3a

2. Invalid

a, b

Fa, ~Ga
Gb, ~Fb

* 1 (∃x)Fx
* 2 (∃x)Gx
[∴ (∃x)(Fx • Gx)
* 3 asm: ~(∃x)(Fx • Gx)
4 ∴ Fa {from 1}
5 ∴ Gb {from 2}
6 ∴ (x)~(Fx • Gx) {from 3}
* 7 ∴ ~(Fa • Ga) {from 6}
8 ∴ ~Ga {from 4 and 7}
* 9 ∴ ~(Fb • Gb) {from 6}
10 ∴ ~Fb {from 5 and 9}

4. Invalid

a

Fa

* 1 (∃x)Fx
[∴ (∃x)~Fx
* 2 asm: ~(∃x)~Fx
3 ∴ Fa {from 1}
4 ∴ (x)Fx {from 2}

6. Invalid

a, b

~Fa, ~Ga
Fb, Gb

1 (x)(Fx ⊃ Gx)
* 2 ~(x)Gx
[∴ (x)~(Fx • Gx)
* 3 asm: ~(x)~(Fx • Gx)
* 4 ∴ (∃x)~Gx {from 2}
* 5 ∴ (∃x)(Fx • Gx) {from 3}
6 ∴ ~Ga {from 4}
* 7 ∴ (Fb • Gb) {from 5}
8 ∴ Fb {from 7}
9 ∴ Gb {from 7}
* 10 ∴ (Fa ⊃ Ga) {from 1}
11 ∴ ~Fa {from 6 and 10}
12 ∴ (Fb ⊃ Gb) {from 1}

7. Invalid

a, b

Fa, ~Ga, ~Ha
Gb, ~Fb, ~Hb

1 (x)((Fx • Gx) ⊃ Hx)
* 2 (∃x)Fx
* 3 (∃x)Gx
[∴ (∃x)Hx
* 4 asm: ~(∃x)Hx
5 ∴ Fa {from 2}

6 ∴ Gb {from 3}
7 ∴ (x)~Hx {from 4}
* 8 ∴ ((Fa • Ga) ⊃ Ha) {from 1}
* 9 ∴ ((Fb • Gb) ⊃ Hb) {from 1}
10 ∴ ~Ha {from 7}
* 11 ∴ ~(Fa • Ga) {from 8 and 10}
12 ∴ ~Ga {from 5 and 11}
13 ∴ ~Hb {from 7}
* 14 ∴ ~(Fb • Gb) {from 9 and 13}
15 ∴ ~Fb {from 6 and 14}

8. Invalid

a, b

Fa, Ga, ~Ha Gb, ~Fb, ~Hb

* 1 (∃x)(Fx ∨ ~Gx)
2 (x)(~Gx ⊃ Hx)
* 3 (∃x)(Fx ⊃ Hx)
[∴ (∃x)Hx
* 4 asm: ~(∃x)Hx
* 5 ∴ (Fa ∨ ~Ga) {from 1}
* 6 ∴ (Fb ⊃ Hb) {from 3}

9. Invalid

~Fa, ~Ga, ~Ha Gb, Hb, ~Fb

* 1 (∃x)~(Fx ∨ Gx)
* 2 (∃x)Hx
* 3 ~(∃x)Fx
[∴ ~(x)(Hx ⊃ Gx)
4 asm: (x)(Hx ⊃ Gx)
* 5 ∴ ~(Fa ∨ Ga) {from 1}
6 ∴ Hb {from 2}
7 ∴ (x)~Fx {from 3}
8 ∴ ~Fa {from 5}
9 ∴ ~Ga {from 5}
* 10 ∴ (Ha ⊃ Ga) {from 4}
11 ∴ ~Ha {from 9 and 10}
* 12 ∴ (Hb ⊃ Gb) {from 4}
13 ∴ Gb {from 6 and 12}
14 ∴ ~Fb {from 7}

5.3b

2. Invalid

a

~Ma, ~Ia

* 1 ~(∃x)(Mx • Ix)
* 2 ~(x)Mx
[∴ (∃x)Ix
* 3 asm: ~(∃x)Ix
4 ∴ (x)~(Mx • Ix) {from 1}
* 5 ∴ (∃x)~Mx {from 2}
6 ∴ (x)~Ix {from 3}
7 ∴ ~Ma {from 5}
8 ∴ ~(Ma • Ia) {from 4}
9 ∴ ~Ia {from 6}

4. Invalid

a, b

Ma, Pa, ~Fa Mb, Fb, ~Pb

* 1 (∃x)(Mx • Px)
* 2 (∃x)(Fx • Mx)
[∴ (∃x)(Fx • Px)
* 3 asm: ~(∃x)(Fx • Px)
* 4 ∴ (Ma • Pa) {from 1}
* 5 ∴ (Fb • Mb) {from 2}
6 ∴ (x)~(Fx • Px) {from 3}
7 ∴ Ma {from 4}
8 ∴ Pa {from 4}
9 ∴ Fb {from 5}
10 ∴ Mb {from 5}
* 11 ∴ ~(Fa • Pa) {from 6}
12 ∴ ~Fa {from 8 and 11}
* 13 ∴ ~(Fb • Pb) {from 6}
14 ∴ ~Pb {from 9 and 13}

6. Invalid

a

Ka, Fa, ~Sa

1 (x)((Kx • Sx) ⊃ Fx)
[∴ (x)((Kx • Fx) ⊃ Sx)
* 2 asm: ~(x)((Kx • Fx) ⊃ Sx)
* 3 ∴ (∃x)~((Kx • Fx) ⊃ Sx) {from 2}
* 4 ∴ ~((Ka • Fa) ⊃ Sa) {from 3}
* 5 ∴ (Ka • Fa) {from 4}
6 ∴ ~Sa {from 4}
7 ∴ Ka {from 5}
8 ∴ Fa {from 5}
9 ∴ ((Ka • Sa) ⊃ Fa) {from 1}

7. Valid

1 (x)(Ex ⊃ Bx)
2 (x)(~Ex ⊃ Bx)
[∴ (x)Bx
* 3 asm: ~(x)Bx
* 4 ∴ (∃x)~Bx {from 3}
5 ∴ ~Ba {from 4}
* 6 ∴ (Ea ⊃ Ba) {from 1}
7 ∴ ~Ea {from 5 and 6}
* 8 ∴ (~Ea ⊃ Ba) {from 2}
9 ∴ Ea {from 5 and 8}
10 ∴ (x)Bx {from 3; 7 contradicts 9}

8. Invalid

a

Aa, Ca

1 (x)(~Cx ⊃ Ax)
[∴ ~(∃x)(Cx • Ax)
* 2 asm: (∃x)(Cx • Ax)
* 3 ∴ (Ca • Aa) {from 2}
4 ∴ Ca {from 3}
5 ∴ Aa {from 3}
6 ∴ (~Ca ⊃ Aa) {from 1}

9. Valid
1 (x)Cx
2 (x)(Cx ⊃ Mx)
[∴ (x)Mx
* 3 ┌ asm: ~(x)Mx
* 4 │ ∴ (∃x)~Mx {from 3}
5 │ ∴ ~Ma {from 4}
6 │ ∴ Ca {from 1}
* 7 │ ∴ (Ca ⊃ Ma) {from 2}
8 └ ∴ ~Ca {from 5 and 7}
9 ∴ (x)Mx {from 3; 6 contradicts 8}

11. Invalid
1 (x)(Tx ⊃ Ex)
[∴ (∃x)(Tx • Ex)
* 2 asm: ~(∃x)(Tx • Ex)
3 ∴ (x)~(Tx • Ex) {from 2}
4 ∴ (Ta ⊃ Ea) {from 1}
5 ∴ ~(Ta • Ea) {from 3}
6 asm: ~Ta {break up 4}

a
$\boxed{\sim Ta}$

12. Valid
* 1 (∃x)Nx
2 (x)(Nx ⊃ Px)
[∴ (∃x)Px
* 3 ┌ asm: ~(∃x)Px
4 │ ∴ Na {from 1}
5 │ ∴ (x)~Px {from 3}
* 6 │ ∴ (Na ⊃ Pa) {from 2}
7 │ ∴ Pa {from 4 and 6}
8 └ ∴ ~Pa {from 5}
9 ∴ (∃x)Px {from 3; 7 contradicts 8}

13. Invalid
1 (x)(Nx ⊃ Px)
[∴ (∃x)(Px • Nx)
* 2 asm: ~(∃x)(Px • Nx)
3 ∴ (x)~(Px • Nx) {from 2}
4 ∴ (Na ⊃ Pa) {from 1}
5 ∴ ~(Pa • Na) {from 3}
6 asm: ~Na {break up 4}

a
$\boxed{\sim Na}$

14. Valid
* 1 ~(∃x)(~Lx • Hx)
[∴ (x)(Hx ⊃ Lx)
* 2 ┌ asm: ~(x)(Hx ⊃ Lx)
3 │ ∴ (x)~(~Lx • Hx) {from 1}
* 4 │ ∴ (∃x)~(Hx ⊃ Lx) {from 2}
* 5 │ ∴ ~(Ha ⊃ La) {from 4}
6 │ ∴ Ha {from 5}
7 │ ∴ ~La {from 5}
* 8 │ ∴ ~(~La • Ha) {from 3}
9 └ ∴ La {from 6 and 8}
10 ∴ (x)(Hx ⊃ Lx) {from 2; 7 contradicts 9}

5.4a

2. (Lg ⊃ (∃x)(Lx • Ex))
4. ((x)(Lx ⊃ Ex) ⊃ (∃x)(Lx • Ex))
6. ((x)Ex ⊃ R)
7. ((∃x)Ex ⊃ R) *or, equivalently,* (x)(Ex ⊃ R)
8. (Lg ⊃ (∃x)Lx)
9. (~(∃x)Ex ⊃ ~(∃x)(Ex • Lx))
11. ((∃x)Lx ⊃ (∃x)Ex)
12. (x)((Cx • Lx) ⊃ Ex)
13. (x)(~Lx ⊃ Ex)
14. ~(x)Ex
16. (Lg ⊃ Eg)
17. (x)(Lx ⊃ Lg) *or, equivalently,* ((∃x)Lx ⊃ Lg)
18. (x)(Lx ⊃ Ex) [This is an exception; "if someone is … then that person is …" just means "all … are …."]
19. (x)(Ex • Lx)

5.5a

2. Valid
1 (x)(Ex ⊃ R)
[∴ ((∃x)Ex ⊃ R)
* 2 ┌ asm: ~((∃x)Ex ⊃ R)
* 3 │ ∴ (∃x)Ex {from 2}
4 │ ∴ ~R {from 2}
5 │ ∴ Ea {from 3}
* 6 │ ∴ (Ea ⊃ R) {from 1}
7 └ ∴ ~Ea {from 4 and 6}
8 ∴ ((∃x)Ex ⊃ R) {from 2; 5 contradicts 7}

4. Valid
* 1 ((∃x)Fx ∨ (∃x)Gx)
[∴ (∃x)(Fx ∨ Gx)
* 2 ┌ asm: ~(∃x)(Fx ∨ Gx)
3 │ ∴ (x)~(Fx ∨ Gx) {from 2}
4 │ ┌ asm: (∃x)Fx {break up 1}
5 │ │ ∴ Fa {from 4}
6 │ │ ∴ ~(Fa ∨ Ga) {from 3}
7 │ └ ∴ ~Fa {from 6}
* 8 │ ∴ ~(∃x)Fx {from 4; 5 contradicts 7}
9 │ ∴ (x)~Fx {from 8}
* 10 │ ∴ (∃x)Gx {from 1 and 8}

11 ∴ Ga {from 10}
* 12 ∴ ~(Fa ∨ Ga) {from 3}
13 ∴ ~Fa {from 12}
14 ∴ ~Ga {from 12}
15 ∴ (∃x)(Fx ∨ Gx) {from 2; 11 contradicts 14}

6. Valid

1 (x)((Fx ∨ Gx) ⊃ Hx)
2 Fm
[∴ Hm
3 asm: ~Hm
* 4 ∴ ((Fm ∨ Gm) ⊃ Hm) {from 1}
5 ∴ ~(Fm ∨ Gm) {from 3 and 4}
6 ∴ ~Fm {from 5}
7 ∴ Hm {from 3; 2 contradicts 6}

7. Invalid

a, j
Ga, ~Ha, ~Fa
Fj, ~Hj, ~Gj

1 Fj
* 2 (∃x)Gx
3 (x)((Fx • Gx) ⊃ Hx)
[∴ (∃x)Hx
* 4 asm: ~(∃x)Hx
5 ∴ Ga {from 2}
6 ∴ (x)~Hx {from 4}
* 7 ∴ ((Fa • Ga) ⊃ Ha) {from 3}
* 8 ∴ ((Fj • Gj) ⊃ Hj) {from 3}
9 ∴ ~Ha {from 6}
* 10 ∴ ~(Fa • Ga) {from 7 and 9}
11 ∴ ~Fa {from 5 and 10}
12 ∴ ~Hj {from 6}
* 13 ∴ ~(Fj • Gj) {from 8 and 12}
14 ∴ ~Gj {from 1 and 13}

8. Valid

* 1 ((∃x)Fx ⊃ (x)Gx)
2 ~Gp
[∴ ~Fp
3 asm: Fp
4 asm: ~(∃x)Fx {break up 1}
5 ∴ (x)~Fx {from 4}
6 ∴ ~Fp {from 5}
7 ∴ (∃x)Fx {from 4; 3 contradicts 6}
8 ∴ (x)Gx {from 1 and 7}
9 ∴ Gp {from 8}
10 ∴ ~Fp {from 3; 2 contradicts 9}

9. Valid

* 1 (∃x)(Fx ∨ Gx)
[∴ ((x)~Gx ⊃ (∃x)Fx)
* 2 asm: ~((x)~Gx ⊃ (∃x)Fx)
* 3 ∴ (Fa ∨ Ga) {from 1}
4 ∴ (x)~Gx {from 2}
* 5 ∴ ~(∃x)Fx {from 2}
6 ∴ (x)~Fx {from 5}
7 ∴ ~Ga {from 4}
8 ∴ Fa {from 3 and 7}
9 ∴ ~Fa {from 6}
10 ∴ ((x)~Gx ⊃ (∃x)Fx) {from 2; 8 contradicts 9}

11. Valid

1 (x)(Ex ⊃ R)
[∴ ((x)Ex ⊃ R)
* 2 asm: ~((x)Ex ⊃ R)
3 ∴ (x)Ex {from 2}
4 ∴ ~R {from 2}
* 5 ∴ (Ea ⊃ R) {from 1}
6 ∴ ~Ea {from 4 and 5}
7 ∴ Ea {from 3}
8 ∴ ((x)Ex ⊃ R) {from 2; 6 contradicts 7}

12. Valid

1 (x)(Fx • Gx)
[∴ ((x)Fx • (x)Gx)
* 2 asm: ~((x)Fx • (x)Gx)
3 asm: ~(x)Fx {break up 2}
4 ∴ (∃x)~Fx {from 3}
5 ∴ ~Fa {from 4}
6 ∴ (Fa • Ga) {from 1}
7 ∴ Fa {from 6}
8 ∴ (x)Fx {from 3; 5 contradicts 7}
* 9 ∴ ~(x)Gx {from 2 and 8}
* 10 ∴ (∃x)~Gx {from 9}
11 ∴ ~Ga {from 10}
* 12 ∴ (Fa • Ga) {from 1}
13 ∴ Fa {from 12}
14 ∴ Ga {from 12}
15 ∴ ((x)Fx • (x)Gx) {from 2; 11 contradicts 14}

13. Valid

* 1 (R ⊃ (x)Ex)
[∴ (x)(R ⊃ Ex)
* 2 asm: ~(x)(R ⊃ Ex)
* 3 ∴ (∃x)~(R ⊃ Ex) {from 2}
* 4 ∴ ~(R ⊃ Ea) {from 3}
5 ∴ R {from 4}
6 ∴ ~Ea {from 4}
7 ∴ (x)Ex {from 1 and 5}
8 ∴ Ea {from 7}

9 ∴ (x)(R ⊃ Ex) {from 2; 6 contradicts 8}

14. Valid

* 1 ((x)Fx ∨ (x)Gx)
[∴ (x)(Fx ∨ Gx)
* 2 asm: ~(x)(Fx ∨ Gx)
* 3 ∴ (∃x)~(Fx ∨ Gx) {from 2}
* 4 ∴ ~(Fa ∨ Ga) {from 3}
5 ∴ ~Fa {from 4}
6 ∴ ~Ga {from 4}
7 asm: (x)Fx {break up 1}
8 ∴ Fa {from 7}
* 9 ∴ ~(x)Fx {from 7; 5 contradicts 8}
10 ∴ (∃x)~Fx {from 9}
11 ∴ (x)Gx {from 1 and 9}
12 ∴ Ga {from 11}
13 ∴ (x)(Fx ∨ Gx) {from 2; 6 contradicts 12}

5.5b

2. Valid

1 (x)Cx
* 2 (G ⊃ (∃x)~Cx)
[∴ ~G
3 asm: G
* 4 ∴ (∃x)~Cx {from 2 and 3}
5 ∴ ~Ca {from 4}
6 ∴ Ca {from 1}
7 ∴ ~G {from 3; 5 contradicts 6}

4. Invalid

a, u
Lu, ~La, ~D

* 1 ((x)Lx ⊃ D)
[∴ (Lu ⊃ D)
* 2 asm: ~(Lu ⊃ D)
3 ∴ Lu {from 2}
4 ∴ ~D {from 2}
* 5 ∴ ~(x)Lx {from 1 and 4}
* 6 ∴ (∃x)~Lx {from 5}
7 ∴ ~La {from 6}

6. Valid

1 (x)(Ex ⊃ (Sx ∨ Fx))
2 ~St
[∴ (~Et ∨ Ft)
* 3 asm: ~(~Et ∨ Ft)
4 ∴ Et {from 3}
5 ∴ ~Ft {from 3}
* 6 ∴ (Et ⊃ (St ∨ Ft)) {from 1}
* 7 ∴ (St ∨ Ft) {from 4 and 6}
8 ∴ Ft {from 2 and 7}
9 ∴ (~Et ∨ Ft) {from 3; 5 contradicts 8}

7. Valid

* 1 ((∃x)Kx ⊃ ~(∃x)Fx)
[∴ ~(∃x)(Kx • Fx)
* 2 asm: (∃x)(Kx • Fx)
* 3 ∴ (Ka • Fa) {from 2}
4 ∴ Ka {from 3}
5 ∴ Fa {from 3}
6 asm: ~(∃x)Kx {break up 1}
7 ∴ (x)~Kx {from 6}
8 ∴ ~Ka {from 7}
9 ∴ (∃x)Kx {from 6; 4 contradicts 8}
*10 ∴ ~(∃x)Fx {from 1 and 9}
11 ∴ (x)~Fx {from 10}
12 ∴ ~Fa {from 11}
13 ∴ ~(∃x)(Kx • Fx) {from 2; 5 contradicts 12}

8. Invalid

a, b
Ta, ~Sa, ~Tb

1 ((x)Tx ⊃ (x)Sx)
[∴ (x)(Tx ⊃ Sx)
* 2 asm: ~(x)(Tx ⊃ Sx)
* 3 ∴ (∃x)~(Tx ⊃ Sx) {from 2}
* 4 ∴ ~(Ta ⊃ Sa) {from 3}
5 ∴ Ta {from 4}
6 ∴ ~Sa {from 4}
**7 asm: ~(x)Tx {break up 1}
**8 ∴ (∃x)~Tx {from 7}
9 ∴ ~Tb {from 8}

9. Invalid

a, b
Ma, Ea, ~Na
Mb, Nb, ~Eb

* 1 ~(∃x)(Ex • Nx)
[∴ (~(∃x)(Mx • Ex) ∨ ~(∃x)(Mx • Nx))
* 2 asm: ~(~(∃x)(Mx • Ex) ∨ ~(∃x)(Mx • Nx))
3 ∴ (x)~(Ex • Nx) {from 1}
* 4 ∴ (∃x)(Mx • Ex) {from 2}
* 5 ∴ (∃x)(Mx • Nx) {from 2}
* 6 ∴ (Ma • Ea) {from 4}
* 7 ∴ (Mb • Nb) {from 5}
8 ∴ Ma {from 6}
9 ∴ Ea {from 6}
10 ∴ Mb {from 7}
11 ∴ Nb {from 7}
*12 ∴ ~(Ea • Na) {from 3}
13 ∴ ~Na {from 9 and 12}
*14 ∴ ~(Eb • Nb) {from 3}
15 ∴ ~Eb {from 11 and 14}

11. Invalid

1 (~(∃x)Nx ⊃ ~(∃x)Cx)
[∴ (x)(Cx ⊃ Nx)
* 2 asm: ~(x)(Cx ⊃ Nx)
* 3 ∴ (∃x)~(Cx ⊃ Nx) {from 2}
* 4 ∴ ~(Ca ⊃ Na) {from 3}
5 ∴ Ca {from 4}
6 ∴ ~Na {from 4}
** 7 asm: (∃x)Nx {break up 1}
8 ∴ Nb {from 7}

a, b
Ca, ~Na, Nb

12. Valid

1 (x)((~Dx • Vx) ⊃ Ox)
2 ~Df
[∴ (Vf ⊃ Of)
* 3 asm: ~(Vf ⊃ Of)
4 ∴ Vf {from 3}
5 ∴ ~Of {from 3}
* 6 ∴ ((~Df • Vf) ⊃ Of) {from 1}
* 7 ∴ ~(~Df • Vf) {from 5 and 6}
8 ∴ ~Vf {from 2 and 7}
9 ∴ (Vf ⊃ Of) {from 3; 4 contradicts 8}

13. Valid

* 1 (~Tw ⊃ (∃x)(Mx • Ix))
* 2 ~(∃x)Ix
[∴ Tw
3 asm: ~Tw
4 ∴ (x)~Ix {from 2}
* 5 ∴ (∃x)(Mx • Ix) {from 1 and 3}
* 6 ∴ (Ma • Ia) {from 5}
7 ∴ Ma {from 6}
8 ∴ Ia {from 6}
9 ∴ ~Ia {from 4}
10 ∴ Tw {from 3; 8 contradicts 9}

14. Valid

1 (x)(Tx ⊃ Cx)
* 2 (Cw ⊃ B)
* 3 (Tw ⊃ ~B)
[∴ ~Tw
4 asm: Tw
5 ∴ ~B {from 3 and 4}
6 ∴ ~Cw {from 2 and 5}
* 7 ∴ (Tw ⊃ Cw) {from 1}
8 ∴ Cw {from 4 and 7}
9 ∴ ~Tw {from 4; 6 contradicts 8}

16. Valid

* 1 ((x)Mx ⊃ (x)(Px ⊃ Cx))
2 Ps
3 ~Cs
[∴ ~(x)Mx
4 asm: (x)Mx
5 ∴ (x)(Px ⊃ Cx) {from 1 and 4}
6 ∴ Ms {from 4}
* 7 ∴ (Ps ⊃ Cs) {from 5}
8 ∴ Cs {from 2 and 7}
9 ∴ ~(x)Mx {from 4; 3 contradicts 8}

17. Invalid

* 1 ((x)Lx ⊃ D)
[∴ ((∃x)Lx ⊃ D)
* 2 asm: ~((∃x)Lx ⊃ D)
* 3 ∴ (∃x)Lx {from 2}
4 ∴ ~D {from 2}
5 ∴ La {from 3}
* 6 ∴ ~(x)Lx {from 1 and 4}
* 7 ∴ (∃x)~Lx {from 6}
8 ∴ ~Lb {from 7}

a, b
La, ~Lb, ~D

18. Valid

1 (x)Mx
2 L
* 3 (L ⊃ (x)(Mx ⊃ Bx))
[∴ (x)Bx
* 4 asm: ~(x)Bx
* 5 ∴ (∃x)~Bx {from 4}
6 ∴ ~Ba {from 5}
7 ∴ (x)(Mx ⊃ Bx) {from 2 and 3}
8 ∴ Ma {from 1}
* 9 ∴ (Ma ⊃ Ba) {from 7}
10 ∴ ~Ma {from 6 and 9}
11 ∴ (x)Bx {from 4; 8 contradicts 10}

19. Valid

1 Be
* 2 (Te • ~Me)
* 3 (De ⊃ Me)
[∴ (∃x)(Bx • ~Dx)
* 4 asm: ~(∃x)(Bx • ~Dx)
5 ∴ Te {from 2}
6 ∴ ~Me {from 2}
7 ∴ (x)~(Bx • ~Dx) {from 4}
8 ∴ ~De {from 3 and 6}
* 9 ∴ ~(Be • ~De) {from 7}
10 ∴ De {from 1 and 9}
11 ∴ (∃x)(Bx • ~Dx) {from 4; 8 contradicts 10}

21. Invalid

1 ((x)Dx ⊃ (x)Bx)
[∴ (x)(Dx ⊃ Bx)
* 2 asm: ~(x)(Dx ⊃ Bx)
* 3 ∴ (∃x)~(Dx ⊃ Bx) {from 2}
* 4 ∴ ~(Da ⊃ Ba) {from 3}
5 ∴ Da {from 4}
6 ∴ ~Ba {from 4}
** 7 asm: ~(x)Dx {break up 1}
** 8 ∴ (∃x)~Dx {from 7}
9 ∴ ~Db {from 8}

a, b

Da, ~Ba, ~Db

22. Valid

1 (x)((Cx • Px) ⊃ Ix)
2 ~Iu
[∴ (Cu ⊃ ~Pu)
* 3 asm: ~(Cu ⊃ ~Pu)
4 ∴ Cu {from 3}
5 ∴ Pu {from 3}
* 6 ∴ ((Cu • Pu) ⊃ Iu) {from 1}
* 7 ∴ ~(Cu • Pu) {from 2 and 6}
8 ∴ ~Pu {from 4 and 7}
9 ∴ (Cu ⊃ ~Pu) {from 3; 5 contradicts 8}

6.1a

2. a=g
4. (∃x)(~x=a • Lx)
6. (La • ~(∃x)(~x=a • Lx))
7. (x)((Lx • ~x=a) ⊃ Ex)
8. ~(∃x)(~x=a • Ex)
9. p=a
11. (∃x)((Ex • Lx) • ~(∃y)(~y=x • (Ey • Ly)))
12. (x)((~x=a • ~x=p) ⊃ Ex)
13. (It ⊃ ~u=t)
14. c=m
16. ~(∃x)(∃y)(~x=y • (Kx • Ky))
17. Bk
18. (∃x)((Kx • ~(∃y)(~x=y • Ky)) • Bx)

6.2a

2. Invalid

1 (a=b ⊃ ~(∃x)Fx)
[∴ (Fa ⊃ ~Fb)
* 2 asm: ~(Fa ⊃ ~Fb)
3 ∴ Fa {from 2}
4 ∴ Fb {from 2}
5 asm: ~a=b {break up 1}

a, b

Fa, ~a=b, Fb

4. Valid

1 ~a=b
2 c=b
[∴ ~a=c
3 asm: a=c
4 ∴ a=b {from 2 and 3}
5 ∴ ~a=c {from 3; 1 contradicts 4}

6. Valid

1 a=b
2 (x)(Fx ⊃ Gx)
3 ~Ga
[∴ ~Fb
4 asm: Fb
5 ∴ Fa {from 1 and 4}
* 6 ∴ (Fa ⊃ Ga) {from 2}
7 ∴ Ga {from 5 and 6}
8 ∴ ~Fb {from 4; 3 contradicts 7}

7. Valid

1 a=b
[∴ (Fa ≡ Fb)
* 2 asm: ~(Fa ≡ Fb)
* 3 ∴ (Fa ∨ Fb) {from 2}
4 ∴ ~(Fa • Fb) {from 2}
5 asm: Fa {break up 3}
6 ∴ Fb {from 1 and 5}
7 ∴ ~Fb {from 4 and 5}
8 ∴ ~Fa {from 5; 6 contradicts 7}
9 ∴ Fb {from 3 and 8}
10 ∴ ~Fb {from 1 and 8}
11 ∴ (Fa ≡ Fb) {from 2; 9 contradicts 10}

8. Valid

1 Fa
[∴ (x)(x=a ⊃ Fx)
* 2 asm: ~(x)(x=a ⊃ Fx)
* 3 ∴ (∃x)~(x=a ⊃ Fx) {from 2}
* 4 ∴ ~(b=a ⊃ Fb) {from 3}
5 ∴ b=a {from 4}
6 ∴ ~Fb {from 4}
7 ∴ ~Fa {from 5 and 6}
8 ∴ (x)(x=a ⊃ Fx) {from 2; 1 contradicts 7}

9. Invalid

[∴ (∃x)(y)y=x
* 1 asm: ~(∃x)(y)y=x
2 ∴ (x)~(y)y=x {from 1}
* 3 ∴ ~(y)y=a {from 2}
4 ∴ (∃y)~y=a {from 3}

a, b

~b=a

5 ∴ ~b=a {from 4}

If we keep going (and drop the universal in line 2 using "b"), we into an endless loop. What we derived so far refutes the argument.

6.2b

2. Valid

* 1 (∃x)Lx
* 2 (∃x)~Lx
[∴ (∃x)(∃y)~x=y
* 3 asm: ~(∃x)(∃y)~x=y
4 ∴ La {from 1}
5 ∴ ~Lb {from 2}
6 ∴ (x)~(∃y)~x=y {from 3}
* 7 ∴ ~(∃y)~a=y {from 6}
8 ∴ (y)a=y {from 7}
9 ∴ a=b {from 8}
10 ∴ Lb {from 4 and 9}
11 ∴ (∃x)(∃y)~x=y {from 3; 5 contradicts 10}

4. Valid

1 l=m
2 Sl
* 3 ~(∃x)(Sx • Bx)
[∴ ~Bm
4 asm: Bm
5 ∴ Bl {from 1 and 4}
6 ∴ (x)~(Sx • Bx) {from 3}
* 7 ∴ ~(Sl • Bl) {from 6}
8 ∴ ~Bl {from 2 and 7}
9 ∴ ~Bm {from 4; 5 contradicts 8}

6. Valid

1 (Ls ⊃ s=p)
2 p=c
[∴ (Ls ⊃ s=c)
3 asm: ~(Ls ⊃ s=c)
4 ∴ (Ls ⊃ s=c) {from 1 and 2}
5 ∴ (Ls ⊃ s=c) {from 3; 3 contradicts 4}

7. Invalid

1 ~j=b
2 Lb
[∴ ~Lj
3 asm: Cj

b, j
| Cb, Cj, ~j=b |

8. Invalid

1 Lp
2 Lb
[∴ (∃x)(∃y)(~x=y • (Lx • Ly))
* 3 asm: ~(∃x)(∃y)(~x=y • (Lx • Ly))
4 ∴ (x)~(∃y)(~x=y • (Lx • Ly)) {from 3}
* 5 ∴ ~(∃y)(~p=y • (Lb • Ly)) {from 4}
6 ∴ (y)~(~p=y • (Lb • Ly)) {from 5}
7 ∴ ~(~p=b • (Lp • Lb)) {from 6}
8 asm: p=b {break up 7}

p, b
| Lp, Lb, p=b |

9. Valid

1 g=a
2 a=b
[∴ g=b
3 asm: ~g=b
4 ∴ ~a=b {from 1 and 3}
5 ∴ g=b {from 3; 2 contradicts 4}

11. Invalid

1 (Rm ∨ Hm)
2 ~Ru
[∴ (Hu ⊃ u=m)
* 3 asm: ~(Hu ⊃ u=m)
4 ∴ Hu {from 3}
5 ∴ ~u=m {from 3}
6 asm: Rm {break up 1}

m, u
| Hu, ~Ru
Rm, ~u=m |

12. Valid

* 1 ((∃x)Cx ⊃ (∃x)Jx)
2 Ci
3 ~Ji
[∴ (∃x)(~x=i • Jx)
* 4 asm: ~(∃x)(~x=i • Jx)
5 ∴ (x)~(~x=i • Jx) {from 4}
6 asm: ~(∃x)Cx {break up 1}
7 ∴ (x)~Cx {from 6}
8 ∴ ~Ci {from 7}
9 ∴ (∃x)Cx {from 6; 2 contradicts 8}
* 10 ∴ (∃x)Jx {from 1 and 9}
11 ∴ Ja {from 10}
* 12 ∴ ~(~a=i • Ja) {from 5}
13 ∴ a=i {from 11 and 12}
14 ∴ Ji {from 11 and 13}
15 ∴ (∃x)(~x=i • Jx) {from 4; 3 contradicts 14}

13. Valid

1 Sd
2 Sn
3 ~d=n
[∴ (∃x)(∃y)(~x=y • (Sx • Sy))
* 4 asm: ~(∃x)(∃y)(~x=y • (Sx • Sy))
5 ∴ (x)~(∃y)(~x=y • (Sx • Sy)) {from 4}
* 6 ∴ ~(∃y)(~d=y • (Sd • Sy)) {from 5}
7 ∴ (y)~(~d=y • (Sd • Sy)) {from 6}

* 8 ∴ ~(~d=n • (Sd • Sn)) {from 7}
* 9 ∴ ~(Sd • Sn) {from 3 and 8}
10 ∴ ~Sn {from 1 and 9}
11 ∴ (∃x)(∃y)(~x=y • (Sx • Sy)) {from 4; 2 contradicts 10}

14. Valid

* 1 ~(∃x)((~x=c • ~x=d) • Kx)
* 2 (∃x)(Kx • Sx)
[∴ (Sc ∨ Sd)
* 3 asm: ~(Sc ∨ Sd)
4 ∴ (x)~((~x=c • ~x=d) • Kx) {from 1}
* 5 ∴ (Ka • Sa) {from 2}
6 ∴ ~Sc {from 3}
7 ∴ ~Sd {from 3}
8 ∴ Ka {from 5}
9 ∴ Sa {from 5}
*10 ∴ ~((~a=c • ~a=d) • Ka) {from 4}
*11 ∴ ~(~a=c • ~a=d) {from 8 and 10}
12 asm: a=c {break up 11}
13 ∴ Sc {from 9 and 12}
14 ∴ ~a=c {from 12; 6 contradicts 13}
15 ∴ a=d {from 11 and 14}
16 ∴ Sd {from 9 and 15}
17 ∴ (Sc ∨ Sd) {from 3; 7 contradicts 16}

16. Invalid

w, c

Pw, Pc, ~c=w

1 Pw
2 ~c=w
[∴ ~Pc
3 asm: Pc

6.3a

2. Cwg
4. (x)(~x=a ⊃ Gax)
6. (x)(y)((Ex • ~Ey) ⊃ Gxy)
7. (Cgw ⊃ Ggw)
8. ~(x)(y)(Cxy ⊃ Gxy)
9. (x)(∃y)Gxy
11. ~(∃x)Cxg
12. (~(∃x)Cxg ⊃ ~(∃x)Cxw)
13. ~(∃x)Cxx
14. (x)(y)(Gxy ⊃ ~Gyx)
16. (∃x)(y)Cxy *or, equivalently,* (∃y)(x)Cyx
17. (∃x)(Ex • (y)(Ey ⊃ Cxy))
18. (x)(~x=g ⊃ Cgx)
19. (x)(y)(z)((Cxy • Cyz) ⊃ Cxz)

6.4a

2. Valid

* 1 (∃x)(y)Lxy
[∴ (∃x)Lxa
* 2 asm: ~(∃x)Lxa
3 ∴ (y)Lby {from 1}
4 ∴ (x)~Lxa {from 2}
5 ∴ Lba {from 3}
6 ∴ ~Lba {from 4}
7 ∴ (∃x)Lxa {from 2; 5 contradicts 6}

4. Invalid

a, b

Lab, Lba, ~Laa

1 (x)(∃y)Lxy
[∴ Laa
2 asm: ~Laa
* 3 ∴ (∃y)Lay {from 1}
4 ∴ Lab {from 3}

Endless loop: add "Lba" to make premise true.

6. Valid

1 (x)(y)(Uxy ⊃ Lxy)
2 (x)(∃y)Uxy
[∴ (x)(∃y)Lxy
* 3 asm: ~(x)(∃y)Lxy
* 4 ∴ (∃x)~(∃y)Lxy {from 3}
* 5 ∴ ~(∃y)Lay {from 4}
6 ∴ (y)~Lay {from 5}
* 7 ∴ (∃y)Uay {from 2}
8 ∴ Uab {from 7}
9 ∴ ~Lab {from 6}
10 ∴ (y)(Uay ⊃ Lay) {from 1}
11 ∴ (Uab ⊃ Lab) {from 10}
12 ∴ Lab {from 8 and 11}
13 ∴ (x)(∃y)Lxy {from 3; 9 contradicts 12}

7. Invalid

a, b

Laa, Lbb
~Lab, ~Lba

1 (x)Lxx
[∴ (∃x)(y)Lxy
* 2 asm: ~(∃x)(y)Lxy
3 ∴ (x)~(y)Lxy {from 2}
* 4 ∴ ~(y)Lay {from 3}
* 5 ∴ (∃y)~Lay {from 4}
6 ∴ ~Lab {from 5}
7 ∴ Laa {from 1}
8 ∴ Lbb {from 1}

Endless loop: add "~Lba" to make conclusion true.

8. Valid

1 (x)Gaxb
[∴ (∃x)(∃y)Gxcy
* 2 asm: ~(∃x)(∃y)Gxcy
3 ∴ (x)~(∃y)Gxcy {from 2}
4 ∴ Gacb {from 1}
* 5 ∴ ~(∃y)Gacy {from 3}
6 ∴ (y)~Gacy {from 5}
7 ∴ ~Gacb {from 6}
8 ∴ (∃x)(∃y)Gxcy {from 2; 4 contradicts 7}

9. Valid

1 (x)(y)Lxy
[∴ (∃x)Lax
* 2 asm: ~(∃x)Lax
3 ∴ (x)~Lax {from 2}
4 ∴ (y)Lay {from 1}
5 ∴ ~Laa {from 3}
6 ∴ Laa {from 4}
7 ∴ (∃x)Lax {from 2; 5 contradicts 6}

11. Invalid

1 (x)Lxx
[∴ (x)(y)(Lxy ⊃ x=y)
* 2 asm: ~(x)(y)(Lxy ⊃ x=y)
* 3 ∴ (∃x)~(y)(Lxy ⊃ x=y) {from 2}
* 4 ∴ ~(y)(Lay ⊃ a=y) {from 3}
* 5 ∴ (∃y)~(Lay ⊃ a=y) {from 4}
* 6 ∴ ~(Lab ⊃ a=b) {from 5}
7 ∴ Lab {from 6}
8 ∴ ~a=b {from 6}
9 ∴ Laa {from 1}
10 ∴ Lbb {from 1}

a, b
Lab, Laa
Lbb, ~a=b

12. Valid

* 1 (∃x)Lxa
2 ~Laa
[∴ (∃x)(~a=x • Lxa)
* 3 asm: ~(∃x)(~a=x • Lxa)
4 ∴ Lba {from 1}
5 ∴ (x)~(~a=x • Lxa) {from 3}
* 6 ∴ ~(~a=b • Lba) {from 5}
7 ∴ a=b {from 4 and 6}
8 ∴ ~Lbb {from 2 and 7}
9 ∴ Lbb {from 4 and 7}
10 ∴ (∃x)(~a=x • Lxa) {from 3; 8 contradicts 9}

13. Invalid

1 (x)(y)(z)((Lxy • Lyz) ⊃ Lxz)
2 (x)(y)(Kxy ⊃ Lyx)
[∴ (x)Lxx
* 3 asm: ~(x)Lxx
* 4 ∴ (∃x)~Lxx {from 3}
5 ∴ ~Laa {from 4}
6 ∴ (y)(z)((Lay • Lyz) ⊃ Laz) {from 1}
7 ∴ (y)(Kay ⊃ Lya) {from 2}
8 ∴ (z)((Laa • Laz) ⊃ Laz) {from 6}
* 9 ∴ (Kaa ⊃ Laa) {from 7}
10 ∴ ~Kaa {from 5 and 9}
* 11 ∴ ((Laa • Laa) ⊃ Laa) {from 8}
12 ∴ ~(Laa • Laa) {from 5 and 11}

a
~Laa
~Kaa

14. Valid

1 (x)Lxa
2 (x)(Lax ⊃ x=b)
[∴ (x)Lxb
* 3 asm: ~(x)Lxb
* 4 ∴ (∃x)~Lxb {from 3}
5 ∴ ~Lcb {from 4}
6 ∴ Laa {from 1}
* 7 ∴ (Laa ⊃ a=b) {from 2}
8 ∴ a=b {from 6 and 7}
9 ∴ (x)Lxb {from 1 and 8}
10 ∴ (x)Lxb {from 3; 3 contradicts 9}

6.4b

2. Valid

* 1 ~(∃x)Cxx
[∴ ~(∃x)(y)Cxy
* 2 asm: (∃x)(y)Cxy
3 ∴ (x)~Cxx {from 1}
4 ∴ (y)Cay {from 2}
5 ∴ ~Caa {from 3}
6 ∴ Caa {from 4}
7 ∴ ~(∃x)(y)Cxy {from 2; 5 contradicts 6}

4. Valid

* 1 (∃y)(x)Dxy
[∴ (x)(∃y)Dxy
* 2 asm: ~(x)(∃y)Dxy
3 ∴ (x)Dxa {from 1}
* 4 ∴ (∃x)~(∃y)Dxy {from 2}
* 5 ∴ ~(∃y)Dby {from 4}
6 ∴ (y)~Dby {from 5}
7 ∴ Dba {from 3}
8 ∴ ~Dba {from 6}

9 ∴ (x)(∃y)Dxy {from 2; 7 contradicts 8}

6. Valid

1 (x)(Fx ⊃ Lrx)
* 2 ~(∃x)(Fx • Lxr)
3 Fj
[∴ (∃x)(Lrx • ~Lxr)
* 4 asm: ~(∃x)(Lrx • ~Lxr)
5 ∴ (x)~(Fx • Lxr) {from 2}
6 ∴ (x)~(Lrx • ~Lxr) {from 4}
* 7 ∴ (Fj ⊃ Lrj) {from 1}
8 ∴ Lrj {from 3 and 7}
* 9 ∴ ~(Fj • Ljr) {from 5}
10 ∴ ~Ljr {from 3 and 9}
* 11 ∴ ~(Lrj • ~Ljr) {from 6}
12 ∴ Ljr {from 8 and 11}
13 ∴ (∃x)(Lrx • ~Lxr) {from 4; 10 contradicts 12}

7. Valid

1 (x)(y)(Cxy ⊃ Bxy)
* 2 ~(∃x)Bxx
[∴ ~(∃x)Cxx
* 3 asm: (∃x)Cxx
4 ∴ (x)~Bxx {from 2}
5 ∴ Caa {from 3}
6 ∴ (y)(Cay ⊃ Bay) {from 1}
7 ∴ ~Baa {from 4}
* 8 ∴ (Caa ⊃ Baa) {from 6}
9 ∴ Baa {from 5 and 8}
10 ∴ ~(∃x)Cxx {from 3; 7 contradicts 9}

8. Valid

1 (x)Lxe
* 2 ~(∃x)(~x=m • Lex)
[∴ e=m
3 asm: ~e=m
4 ∴ (x)~(~x=m • Lex) {from 2}
5 ∴ Lee {from 1}
* 6 ∴ ~(~e=m • Lee) {from 4}
7 ∴ ~Lee {from 3 and 6}
8 ∴ e=m {from 3; 5 contradicts 7}

9. Invalid

* 1 ~(x)(y)Lxy
[∴ ~(x)Lxu
2 asm: (x)Lxu
* 3 ∴ (∃x)~(y)Lxy {from 1}
* 4 ∴ ~(y)Lay {from 3}
* 5 ∴ (∃y)~Lay {from 4}
6 ∴ ~Lab {from 5}
7 ∴ Lau {from 2}
8 ∴ Lbu {from 2}
9 ∴ Luu {from 2}

a, b, u

~Lab, Lau
Lbu, Luu

11. Valid

1 (x)(Sax ≡ ~Sxx)
[∴ R
2 asm: ~R
* 3 ∴ (Saa ≡ ~Saa) {from 1}
* 4 ∴ (Saa ⊃ ~Saa) {from 3}
5 ∴ (~Saa ⊃ Saa) {from 3}
6 asm: ~Saa {break up 4}
7 ∴ Saa {from 5 and 6}
8 ∴ Saa {from 6; 6 contradicts 7}
9 ∴ ~Saa {from 4 and 8}
10 ∴ R {from 2; 8 contradicts 9}

12. Valid

* 1 ~(∃x)Hxx
2 (x)(Lx ⊃ Hix)
[∴ ~Li
3 asm: Li
4 ∴ (x)~Hxx {from 1}
* 5 ∴ (Li ⊃ Hii) {from 2}
6 ∴ Hii {from 3 and 5}
7 ∴ ~Hii {from 4}
8 ∴ ~Li {from 3; 6 contradicts 7}

13. Valid

1 (x)(~x=j ⊃ Ljx)
2 Ij
3 r=m
4 ~Im
[∴ Ljr
5 asm: ~Ljr
6 ∴ ~Ljm {from 3 and 5}
* 7 ∴ (~m=j ⊃ Ljm) {from 1}
8 ∴ m=j {from 6 and 7}
9 ∴ ~Ij {from 4 and 8}
10 ∴ Ljr {from 5; 2 contradicts 9}

14. Valid

* 1 (Lrl ∨ Lrc)
2 (x)(~Ix ⊃ ~Lrx)
3 ~Ic
[∴ Lrl
4 asm: ~Lrl
5 ∴ Lrc {from 1 and 4}
* 6 ∴ (~Ic ⊃ ~Lrc) {from 2}
7 ∴ ~Lrc {from 3 and 6}
8 ∴ Lrl {from 4; 5 contradicts 7}

16. Invalid

a, b

Lab, Lba
~Laa, ~Lbb

1 (x)(∃y)Lxy
[∴ (∃x)Lxx
* 2 asm: ~(∃x)Lxx
3 ∴ (x)~Lxx {from 2}
4 ∴ ~Laa {from 3}
* 5 ∴ (∃y)Lay {from 1}
6 ∴ Lab {from 5}

Endless loop: add "Lba" to make the premise true and "~Lbb" to make the conclusion false.

17. Valid

* 1 ~(∃x)Cxx
2 Cbp
[∴ ~b=p
3 asm: b=p
4 ∴ (x)~Cxx {from 1}
5 ∴ Cpp {from 2 and 3}
6 ∴ ~Cpp {from 4}
7 ∴ ~b=p {from 3; 5 contradicts 6}

18. Valid

1 (x)(Cx ⊃ (∃y)Eyx)
2 Ce
* 3 ((∃x)Exe ⊃ (∃x)(Nx • Exe))
[∴ (∃x)(Nx • Exe)
* 4 asm: ~(∃x)(Nx • Exe)
5 ∴ (x)~(Nx • Exe) {from 4}
* 6 ∴ ~(∃x)Exe {from 3 and 4}
7 ∴ (x)~Exe {from 6}
* 8 ∴ (Ce ⊃ (∃y)Eye) {from 1}
* 9 ∴ (∃y)Eye {from 2 and 8}
10 ∴ Eae {from 9}
11 ∴ (Ca ⊃ (∃y)Eya) {from 1}
* 12 ∴ ~(Na • Eae) {from 5}
13 ∴ ~Na {from 10 and 12}
14 ∴ ~(Ne • Eee) {from 5}
15 ∴ ~Eae {from 7}
16 ∴ (∃x)(Nx • Exe) {from 4; 10 contradicts 15}

19. Valid

1 t=m
[∴ (~Lum ⊃ ~Lut)
* 2 asm: ~(~Lum ⊃ ~Lut)
3 ∴ ~Lum {from 2}
4 ∴ Lut {from 2}
5 ∴ ~Lut {from 1 and 3}
6 ∴ (~Lum ⊃ ~Lut) {from 2; 4 contradicts 5}

21. Valid

1 Gf
2 Ek
3 Cfk
4 ~Pfk
[∴ ~(x)(Gx ⊃ (y)((Ey • Cxy) ⊃ Pxy))
5 asm: (x)(Gx ⊃ (y)((Ey • Cxy) ⊃ Pxy))
* 6 ∴ (Gf ⊃ (y)((Ey • Cfy) ⊃ Pfy)) {from 5}
7 ∴ (y)((Ey • Cfy) ⊃ Pfy) {from 1 and 6}
* 8 ∴ ((Ek • Cfk) ⊃ Pfk) {from 7}
* 9 ∴ ~(Ek • Cfk) {from 4 and 8}
10 ∴ ~Cfk {from 2 and 9}
11 ∴ ~(x)(Gx ⊃ (y)((Ey • Cxy) ⊃ Pxy)) {from 5; 3 contradicts 10}

22. Invalid

a, b,
t´, t´´

Ca, Cb
Eat´´, ~Eat´
Ebt´, ~Ebt´´

1 (x)(Cx ⊃ (∃t)~Ext)
[∴ ((x)Cx ⊃ (∃t)(x)~Ext)
* 2 asm: ~((x)Cx ⊃ (∃t)(x)~Ext)
3 ∴ (x)Cx {from 2}
* 4 ∴ ~(∃t)(x)~Ext {from 2}
5 ∴ (t)~(x)~Ext {from 4}
6 ∴ Ca {from 3}
* 7 ∴ (Ca ⊃ (∃t)~Eat) {from 1}
* 8 ∴ (∃t)~Eat {from 6 and 7}
9 ∴ ~Eat´ {from 8}
* 10 ∴ ~(x)~Ext´ {from 5}
* 11 ∴ (∃x)Ext´ {from 10}
12 ∴ Ebt´ {from 11}
13 ∴ Cb {from 3}
* 14 ∴ (Cb ⊃ (∃t)~Ebt) {from 1}
* 15 ∴ (∃t)~Ebt {from 13 and 14}
16 ∴ ~Ebt´´ {from 15}

Endless loop: add "Eat´´" and "Ebt´" to make conclusion false. In this world, we have two contingent things and two times; each contingent thing exists at one of the times but not the other – which makes the premise true but the conclusion false.

23. Valid

* 1 ((x)Cx ⊃ (∃t)(x)~Ext)
* 2 ((∃t)(x)~Ext ⊃ ~(∃x)Exn)
* 3 (∃x)Exn
4 (x)(~Cx ⊃ Nx)
[∴ (∃x)Nx
* 5 asm: ~(∃x)Nx
6 ∴ (x)~Nx {from 5}
* 7 ∴ ~(∃t)(x)~Ext {from 2 and 3}
* 8 ∴ ~(x)Cx {from 1 and 7}

* 9 ∴ (∃x)~Cx {from 8}
10 ∴ ~Cb {from 9}
*11 ∴ (~Cb ⊃ Nb) {from 4}
12 ∴ Nb {from 10 and 11}
13 ∴ ~Nb {from 6}
14 ∴ (∃x)Nx {from 5; 12 contradicts 13}

24. Valid

* 1 (D ⊃ (∃y)(Sy • (x)(Cyx ≡ ~Cxx)))
[∴ ~D
2 asm: D
* 3 ∴ (∃y)(Sy • (x)(Cyx ≡ ~Cxx)) {from 1 and 2}
* 4 ∴ (Sa • (x)(Cax ≡ ~Cxx)) {from 3}
5 ∴ Sa {from 4}
6 ∴ (x)(Cax ≡ ~Cxx) {from 4}
* 7 ∴ (Caa ≡ ~Caa) {from 6}
* 8 ∴ (Caa ⊃ ~Caa) {from 7}
9 ∴ (~Caa ⊃ Caa) {from 7}
10 asm: ~Caa {break up 8}
11 ∴ Caa {from 9 and 10}
12 ∴ Caa {from 10; 10 contradicts 11}
13 ∴ ~Caa {from 8 and 12}
14 ∴ ~D {from 2; 12 contradicts 13}

7.1a

2. ~◇G
4. □~M
6. ~□(P ⊃ R)
7. □(~P ⊃ ~R)
8. (◇R ⊃ ◇P)
9. G
11. ~◇E
12. ◇~E
13. (R ⊃ ◇R)
14. ◇(M • E)
16. □(G ⊃ ~E)
17. Ambiguous: (G ⊃ ~◇E) or □(G ⊃ ~E); the first also could be written as "(G ⊃ □~E)."
18. (□M ⊃ E)
19. □(G ⊃ ~◇E); this also could be written as: "□(G ⊃ □~E)."
21. (□G ∨ □M)
22. (◇R • ◇~R)
23. (R • ◇~R)
24. □(R ⊃ E)
26. Ambiguous: (R ⊃ □E) or □(R ⊃ E)
27. □◇M
28. ~(G • ◇~G)
29. Ambiguous: (G ⊃ □G) or □(G ⊃ G)

7.2a

2. Valid

1 A
[∴ ◇A
* 2 asm: ~◇A
3 ∴ □~A {from 2}
4 ∴ ~A {from 3}
5 ∴ ◇A {from 2; 1 contradicts 4}

4. Valid

1 □(A ∨ ~B)
* 2 ~□A
[∴ ◇~B
* 3 asm: ~◇~B
* 4 ∴ ◇~A {from 2}
5 ∴ □B {from 3}
6 W ∴ ~A {from 4}
* 7 W ∴ (A ∨ ~B) {from 1}
8 W ∴ ~B {from 6 and 7}
9 W ∴ B {from 5}
10 ∴ ◇~B {from 3; 8 contradicts 9}

6. Valid

* 1 (A ⊃ □B)
* 2 ◇~B
[∴ ◇~A
* 3 asm: ~◇~A
4 W ∴ ~B {from 2}
5 ∴ □A {from 3}
6 ∴ A {from 5}
7 ∴ □B {from 1 and 6}
8 W ∴ B {from 7}
9 ∴ ◇~A {from 3; 4 contradicts 8}

7. Valid

* 1 ~◇(A • B)
* 2 ◇A
[∴ ~□B
3 asm: □B
4 ∴ □~(A • B) {from 1}
5 W ∴ A {from 2}
6 W ∴ B {from 3}
* 7 W ∴ ~(A • B) {from 4}
8 W ∴ ~B {from 5 and 7}
9 ∴ ~□B {from 3; 6 contradicts 8}

8. Valid

1 ☐A
[∴ ◇A
* 2 asm: ~◇A
3 ∴ ☐~A {from 2}
4 ∴ A {from 1}
5 ∴ ~A {from 3}
6 ∴ ◇A {from 2; 4 contradicts 5}

9. Valid

1 ☐A
* 2 ~☐B
[∴ ~☐(A ⊃ B)
3 asm: ☐(A ⊃ B)
* 4 ∴ ◇~B {from 2}
5 W ∴ ~B {from 4}
6 W ∴ A {from 1}
* 7 W ∴ (A ⊃ B) {from 3}
8 W ∴ ~A {from 5 and 7}
9 ∴ ~☐(A ⊃ B) {from 3; 6 contradicts 8}

7.2b

2. Valid

1 ☐(~D ⊃ N)
2 ☐(N ⊃ D)
[∴ ☐(~D ⊃ D)
* 3 asm: ~☐(~D ⊃ D)
* 4 ∴ ◇~(~D ⊃ D) {from 3}
5 W ∴ ~(~D ⊃ D) {from 4}
6 W ∴ ~D {from 5}
* 7 W ∴ (~D ⊃ N) {from 1}
8 W ∴ N {from 6 and 7}
* 9 W ∴ (N ⊃ D) {from 2}
10 W ∴ ~N {from 6 and 9}
11 ∴ ☐(~D ⊃ D) {from 3; 8 contradicts 10}

4. Valid

1 G
2 E
[∴ ◇(G • E)
* 3 asm: ~◇(G • E)
4 ∴ ☐~(G • E) {from 3}
* 5 ∴ ~(G • E) {from 4}
6 ∴ ~E {from 1 and 5}
7 ∴ ◇(G • E) {from 3; 2 contradicts 6}

6. Valid

* 1 ◇(G • (E • R))
[∴ ◇(G • E)
* 2 asm: ~◇(G • E)
* 3 W ∴ (G • (E • R)) {from 1}
4 ∴ ☐~(G • E) {from 2}
5 W ∴ G {from 3}
* 6 W ∴ (E • R) {from 3}
7 W ∴ E {from 6}
8 W ∴ R {from 6}
* 9 W ∴ ~(G • E) {from 4}
10 W ∴ ~E {from 5 and 9}
11 ∴ ◇(G • E) {from 2; 7 contradicts 10}

7. Valid

1 O
2 ◇F
* 3 ((◇F • O) ⊃ ◇B)
[∴ ◇B
4 asm: ~◇B
* 5 ∴ ~(◇F • O) {from 3 and 4}
6 ∴ ~◇F {from 1 and 5}
7 ∴ ◇B {from 4; 2 contradicts 6}

8. Valid

* 1 ~◇(B • F)
2 ☐(G ⊃ B)
3 ☐(G ⊃ F)
[∴ ~◇G
* 4 asm: ◇G
5 ∴ ☐~(B • F) {from 1}
6 W ∴ G {from 4}
* 7 W ∴ (G ⊃ B) {from 2}
8 W ∴ B {from 6 and 7}
* 9 W ∴ (G ⊃ F) {from 3}
10 W ∴ F {from 6 and 9}
* 11 W ∴ ~(B • F) {from 5}
12 W ∴ ~F {from 8 and 11}
13 ∴ ~◇G {from 4; 10 contradicts 12}

9. Valid

1 ☐(S ⊃ L)
2 ☐(C ⊃ ~L)
[∴ ~◇(S • C)
* 3 asm: ◇(S • C)
* 4 W ∴ (S • C) {from 3}
5 W ∴ S {from 4}
6 W ∴ C {from 4}
* 7 W ∴ (S ⊃ L) {from 1}
8 W ∴ L {from 5 and 7}
* 9 W ∴ (C ⊃ ~L) {from 2}
10 W ∴ ~L {from 6 and 9}
11 ∴ ~◇(S • C) {from 3; 8 contradicts 10}

11. Valid

1 □A
* 2 ~□X
3 □(S ⊃ X)
[∴ ~□(A ⊃ S)
4 ┌ asm: □(A ⊃ S)
* 5 │ ∴ ◇~X {from 2}
6 │ W ∴ ~X {from 5}
7 │ W ∴ A {from 1}
* 8 │ W ∴ (A ⊃ S) {from 4}
9 │ W ∴ S {from 7 and 8}
* 10 │ W ∴ (S ⊃ X) {from 3}
11 └ W ∴ ~S {from 6 and 10}
12 ∴ ~□(A ⊃ S) {from 4; 9 contradicts 11}

12. Valid

1 □((P • ~R) ⊃ C)
* 2 ~◇C
3 □P
[∴ □R
* 4 ┌ asm: ~□R
5 │ ∴ □~C {from 2}
* 6 │ ∴ ◇~R {from 4}
7 │ W ∴ ~R {from 6}
* 8 │ W ∴ ((P • ~R) ⊃ C) {from 1}
9 │ W ∴ P {from 3}
10 │ W ∴ ~C {from 5}
* 11 │ W ∴ ~(P • ~R) {from 8 and 10}
12 └ W ∴ ~P {from 7 and 11}
13 ∴ □R {from 4; 9 contradicts 12}

13. Valid

* 1 (I ⊃ □(L ⊃ G))
* 2 ◇(L • ~G)
[∴ ~I
3 ┌ asm: I
* 4 │ W ∴ (L • ~G) {from 2}
5 │ W ∴ L {from 4}
6 │ W ∴ ~G {from 4}
7 │ ∴ □(L ⊃ G) {from 1 and 3}
* 8 │ W ∴ (L ⊃ G) {from 7}
9 └ W ∴ G {from 5 and 8}
10 ∴ ~I {from 3; 6 contradicts 9}

14. Valid

1 □(S ⊃ (K • (A • ~D)))
2 □(K ⊃ W)
3 □((W • A) ⊃ D)
[∴ ~◇S
* 4 ┌ asm: ◇S
5 │ W ∴ S {from 4}
* 6 │ W ∴ (S ⊃ (K • (A • ~D))) {from 1}
* 7 │ W ∴ (K • (A • ~D)) {from 5 and 6}
8 │ W ∴ K {from 7}
* 9 │ W ∴ (A • ~D) {from 7}
10 │ W ∴ A {from 9}
11 │ W ∴ ~D {from 9}
* 12 │ W ∴ (K ⊃ W) {from 2}
13 │ W ∴ W {from 8 and 12}
* 14 │ W ∴ ((W • A) ⊃ D) {from 3}
* 15 │ W ∴ ~(W • A) {from 11 and 14}
16 └ W ∴ ~W {from 10 and 15}
17 ∴ ~◇S {from 4; 13 contradicts 16}

7.3a

2. Invalid

	A
W	~A

1 A
[∴ □A
* 2 asm: ~□A
* 3 ∴ ◇~A {from 2}
4 W ∴ ~A {from 3}

4. Invalid

	B, ~A
W	A, ~B

1 □(A ⊃ ~B)
2 B
[∴ □~A
* 3 asm: ~□~A
* 4 ∴ ◇A {from 3}
5 W ∴ A {from 4}
* 6 ∴ (A ⊃ ~B) {from 1}
7 ∴ ~A {from 2 and 6}
* 8 W ∴ (A ⊃ ~B) {from 1}
9 W ∴ ~B {from 5 and 8}

6. Invalid

W	A, B
WW	~A, ~B

* 1 ◇A
* 2 ~□B
[∴ ~□(A ⊃ B)
3 asm: □(A ⊃ B)
4 W ∴ A {from 1}
* 5 ∴ ◇~B {from 2}
6 WW ∴ ~B {from 5}
* 7 W ∴ (A ⊃ B) {from 3}
8 W ∴ B {from 4 and 7}
* 9 WW ∴ (A ⊃ B) {from 3}
10 WW ∴ ~A {from 6 and 9}

7. Invalid

	B, C, ~A
W	A, C, ~B

1 □(C ⊃ (A ∨ B))

* 2 (~A · ◇~B)
[∴ ◇~C
* 3 asm: ~◇~C
4 ∴ ~A {from 2}
* 5 ∴ ◇~B {from 2}
6 ∴ ☐C {from 3}
7 W ∴ ~B {from 5}
* 8 ∴ (C ⊃ (A ∨ B)) {from 1}
* 9 W ∴ (C ⊃ (A ∨ B)) {from 1}
10 ∴ C {from 6}
*11 ∴ (A ∨ B) {from 8 and 10}
12 ∴ B {from 4 and 11}
13 W ∴ C {from 6}
*14 W ∴ (A ∨ B) {from 9 and 13}
15 W ∴ A {from 7 and 14}

8. Invalid

W	A, B
WW	~A, ~B

* 1 ◇A
* 2 ~☐B
[∴ ~☐(A ⊃ B)
3 asm: ☐(A ⊃ B)
4 W ∴ A {from 1}
* 5 ∴ ◇~B {from 2}
6 WW ∴ ~B {from 5}
* 7 W ∴ (A ⊃ B) {from 3}
8 W ∴ B {from 4 and 7}
* 9 WW ∴ (A ⊃ B) {from 3}
10 WW ∴ ~A {from 6 and 9}

9. Invalid

W	A, ~B, ~C
WW	B, ~A, ~C

1 ☐((A · B) ⊃ C)
* 2 ◇A
* 3 ◇B
[∴ ◇C
* 4 asm: ~◇C
5 W ∴ A {from 2}
6 WW ∴ B {from 3}
7 ∴ ☐~C {from 4}
* 8 W ∴ ((A · B) ⊃ C) {from 1}
* 9 WW ∴ ((A · B) ⊃ C) {from 1}
10 W ∴ ~C {from 7}
*11 W ∴ ~(A · B) {from 8 and 10}
12 W ∴ ~B {from 5 and 11}
13 WW ∴ ~C {from 7}
*14 WW ∴ ~(A · B) {from 9 and 13}
15 WW ∴ ~A {from 6 and 14}

7.3b

2. Invalid

	K, ~M
W	M, ~K

1 K
* 2 ◇M
[∴ ◇(K · M)
* 3 asm: ~◇(K · M)
4 W ∴ M {from 2}
5 ∴ ☐~(K · M) {from 3}
* 6 ∴ ~(K · M) {from 5}
7 ∴ ~M {from 1 and 6}
* 8 W ∴ ~(K · M) {from 5}
9 W ∴ ~K {from 4 and 8}

4. The first premise is ambiguous. The box-inside form gives a valid argument (but with a false or questionable first premise); the box-outside form is invalid.

Valid

* 1 (S ⊃ ☐~M)
2 S
[∴ ☐~M
3 asm: ~☐~M
4 ∴ ☐~M {from 1 and 2}
5 ∴ ☐~M {from 3; 3 contradicts 4}

Invalid

	S, ~M
W	M, ~S

1 ☐(S ⊃ ~M)
2 S
[∴ ☐~M
* 3 asm: ~☐~M
* 4 ∴ ◇M {from 3}
5 W ∴ M {from 4}
* 6 ∴ (S ⊃ ~M) {from 1}
7 ∴ ~M {from 2 and 6}
* 8 W ∴ (S ⊃ ~M) {from 1}
9 W ∴ ~S {from 5 and 8}

6. Valid

* 1 (☐(~C ⊃ L) ⊃ F)
* 2 (F ⊃ I)
* 3 (I ⊃ ☐(C ⊃ L))
[∴ (◇~L ⊃ ~☐(~C ⊃ L))
* 4 asm: ~(◇~L ⊃ ~☐(~C ⊃ L))
* 5 ∴ ◇~L {from 4}
6 ∴ ☐(~C ⊃ L) {from 4}
7 W ∴ ~L {from 5}
8 ∴ F {from 1 and 6}
9 ∴ I {from 2 and 8}
10 ∴ ☐(C ⊃ L) {from 3 and 9}

11 ∴ (~C ⊃ L) {from 6}
* 12 W ∴ (~C ⊃ L) {from 6}
13 W ∴ C {from 7 and 12}
14 ∴ (C ⊃ L) {from 10}
* 15 W ∴ (C ⊃ L) {from 10}
16 W ∴ ~C {from 7 and 15}
17 ∴ (◇~L ⊃ ~□(~C ⊃ L)) {from 4; 13 contradicts 16}

7. The first premise is ambiguous. The box-inside form gives a valid argument (but with a false or questionable first premise); the box-outside form is invalid.

Valid

* 1 (M ⊃ □~B)
* 2 ◇B
[∴ ~M
3 asm: M
4 W ∴ B {from 2}
5 ∴ □~B {from 1 and 3}
6 W ∴ ~B {from 5}
7 ∴ ~M {from 3; 4 contradicts 6}

Invalid

1 □(M ⊃ ~B)
* 2 ◇B
[∴ ~M
3 asm: M
4 W ∴ B {from 2}
* 5 ∴ (M ⊃ ~B) {from 1}
6 ∴ ~B {from 3 and 5}
* 7 W ∴ (M ⊃ ~B) {from 1}
8 W ∴ ~M {from 4 and 7}

	M, ~B
W	B, ~M

8. The first premise is ambiguous. The box-inside form gives a valid argument (but with a false or questionable first premise); the box-outside form is invalid.

Valid

* 1 (K ⊃ □~M)
* 2 ◇M
[∴ ~K
3 asm: K
4 W ∴ M {from 2}
5 ∴ □~M {from 1 and 3}
6 W ∴ ~M {from 5}
7 ∴ ~K {from 3; 4 contradicts 6}

Invalid

1 □(K ⊃ ~M)
* 2 ◇M
[∴ ~K
3 asm: K
4 W ∴ M {from 2}
* 5 ∴ (K ⊃ ~M) {from 1}
6 ∴ ~M {from 3 and 5}
* 7 W ∴ (K ⊃ ~M) {from 1}
8 W ∴ ~K {from 4 and 7}

	K, ~M
W	M, ~K

9. Valid (but some would question the step from 8 to 9 – see Section 8.1)

1 □(N ⊃ □N)
* 2 ◇N
[∴ □N
* 3 asm: ~□N
4 W ∴ N {from 2}
* 5 ∴ ◇~N {from 3}
6 WW ∴ ~N {from 5}
* 7 W ∴ (N ⊃ □N) {from 1}
8 W ∴ □N {from 4 and 7}
9 WW ∴ N {from 8}
10 ∴ □N {from 3; 6 contradicts 9}

11. Invalid

* 1 ◇(D • A)
* 2 ◇(A • P)
[∴ ◇(D • P)
* 3 asm: ~◇(D • P)
* 4 W ∴ (D • A) {from 1}
* 5 WW ∴ (A • P) {from 2}
6 ∴ □~(D • P) {from 3}
7 W ∴ D {from 4}
8 W ∴ A {from 4}
9 WW ∴ A {from 5}
10 WW ∴ P {from 5}
* 11 W ∴ ~(D • P) {from 6}
12 W ∴ ~P {from 7 and 11}
* 13 WW ∴ ~(D • P) {from 6}
14 WW ∴ ~D {from 10 and 13}

W	A, D, ~P
WW	A, P, ~D

12. Valid

* 1 (J ⊃ □(T ⊃ C))
2 □(C ⊃ B)
* 3 ◇(T • ~B)
[∴ ~J
4 asm: J
* 5 W ∴ (T • ~B) {from 3}
6 W ∴ T {from 5}
7 W ∴ ~B {from 5}
8 ∴ □(T ⊃ C) {from 1 and 4}

* 9 W ∴ (C ⊃ B) {from 2}
10 W ∴ ~C {from 7 and 9}
*11 W ∴ (T ⊃ C) {from 8}
12 W ∴ C {from 6 and 11}
13 ∴ ~J {from 4; 10 contradicts 12}

13. Valid

* 1 (□(D ⊃ P) ⊃ I)
* 2 ~◇I
[∴ ◇(D • ~P)
* 3 asm: ~◇(D • ~P)
4 ∴ □~I {from 2}
5 ∴ □~(D • ~P) {from 3}
6 ∴ ~I {from 4}
* 7 ∴ ~□(D ⊃ P) {from 1 and 6}
* 8 ∴ ◇~(D ⊃ P) {from 7}
* 9 W ∴ ~(D ⊃ P) {from 8}
10 W ∴ D {from 9}
11 W ∴ ~P {from 9}
12 W ∴ ~I {from 4}
*13 W ∴ ~(D • ~P) {from 5}
14 W ∴ P {from 10 and 13}
15 ∴ ◇(D • ~P) {from 3; 11 contradicts 14}

14. The second premise is ambiguous. The box-inside form gives a valid argument (but with a false or questionable second premise); the box-outside form is invalid.

Valid

1 K
* 2 (K ⊃ □D)
* 3 (□D ⊃ ~F)
[∴ ~F
4 asm: F
5 ∴ □D {from 1 and 2}
6 ∴ ~□D {from 3 and 4}
7 ∴ ~F {from 4; 5 contradicts 6}

Invalid

	K, D, F
W	~K, ~D

1 K
2 □(K ⊃ D)
* 3 (□D ⊃ ~F)
[∴ ~F
4 asm: F
* 5 ∴ ~□D {from 3 and 4}
* 6 ∴ ◇~D {from 5}
7 W ∴ ~D {from 6}
* 8 ∴ (K ⊃ D) {from 2}
9 ∴ D {from 1 and 8}
*10 W ∴ (K ⊃ D) {from 2}
11 W ∴ ~K {from 7 and 10}

16. Valid

1 □(B ⊃ A)
* 2 ~◇A
[∴ ~◇B
* 3 asm: ◇B
4 ∴ □~A {from 2}
5 W ∴ B {from 3}
* 6 W ∴ (B ⊃ A) {from 1}
7 W ∴ A {from 5 and 6}
8 W ∴ ~A {from 4}
9 ∴ ~◇B {from 3; 7 contradicts 8}

17. The first premise is ambiguous. The box-outside form (which better represents Kant's argument) gives a valid argument with plausible premises; the box-inside form is invalid and has a false or questionable first premise.

Valid

1 □(E ⊃ T)
2 □(T ⊃ C)
[∴ (◇E ⊃ ◇C)
* 3 asm: ~(◇E ⊃ ◇C)
* 4 ∴ ◇E {from 3}
* 5 ∴ ~◇C {from 3}
6 W ∴ E {from 4}
7 ∴ □~C {from 5}
* 8 W ∴ (E ⊃ T) {from 1}
9 W ∴ T {from 6 and 8}
*10 W ∴ (T ⊃ C) {from 2}
11 W ∴ C {from 9 and 10}
12 W ∴ ~C {from 7}
13 ∴ (◇E ⊃ ◇C) {from 3; 11 contradicts 12}

Invalid

	~C, ~T, ~E
W	E, ~C, ~T

1 (E ⊃ □T)
2 □(T ⊃ C)
[∴ (◇E ⊃ ◇C)
* 3 asm: ~(◇E ⊃ ◇C)
* 4 ∴ ◇E {from 3}
* 5 ∴ ~◇C {from 3}
6 W ∴ E {from 4}
7 ∴ □~C {from 5}
* 8 ∴ (T ⊃ C) {from 2}
* 9 W ∴ (T ⊃ C) {from 2}
10 ∴ ~C {from 7}
11 ∴ ~T {from 8 and 10}
12 W ∴ ~C {from 7}
13 W ∴ ~T {from 9 and 12}

14 asm: ~E {break up 1}

18. Invalid

	A, B
W	~A, ~B

1 ☐(A ⊃ B)
[∴ (A ⊃ ☐B)
* 2 asm: ~(A ⊃ ☐B)
3 ∴ A {from 2}
* 4 ∴ ~☐B {from 2}
* 5 ∴ ◇~B {from 4}
6 W ∴ ~B {from 5}
* 7 ∴ (A ⊃ B) {from 1}
8 ∴ B {from 3 and 7}
* 9 W ∴ (A ⊃ B) {from 1}
10 W ∴ ~A {from 6 and 9}

19. Valid

1 ☐(M ⊃ ~E)
2 ☐(M ⊃ E)
[∴ ~◇M
* 3 asm: ◇M
4 W ∴ M {from 3}
* 5 W ∴ (M ⊃ ~E) {from 1}
6 W ∴ ~E {from 4 and 5}
* 7 W ∴ (M ⊃ E) {from 2}
8 W ∴ E {from 4 and 7}
9 ∴ ~◇M {from 3; 6 contradicts 8}

21. The first premise is ambiguous. The box-inside form is valid, while the box-outside form is invalid.

Valid

* 1 (P ⊃ ☐S)
* 2 ◇~S
[∴ ~P
3 asm: P
4 W ∴ ~S {from 2}
5 ∴ ☐S {from 1 and 3}
6 W ∴ S {from 5}
7 ∴ ~P {from 3; 4 contradicts 6}

Invalid

	P, S
W	~P, ~S

1 ☐(P ⊃ S)
* 2 ◇~S
[∴ ~P
3 asm: P
4 W ∴ ~S {from 2}
* 5 ∴ (P ⊃ S) {from 1}
6 ∴ S {from 3 and 5}
* 7 W ∴ (P ⊃ S) {from 1}
8 W ∴ ~P {from 4 and 7}

22. Invalid

	D
W	S, ~A
WW	S, A

* 1 ~☐(S ⊃ A)
* 2 (D ⊃ ◇(S • A))
[∴ ~D
3 asm: D
* 4 ∴ ◇~(S ⊃ A) {from 1}
* 5 W ∴ ~(S ⊃ A) {from 4}
6 W ∴ S {from 5}
7 W ∴ ~A {from 5}
* 8 ∴ ◇(S • A) {from 2 and 3}
* 9 WW ∴ (S • A) {from 8}
10 WW ∴ S {from 9}
11 WW ∴ A {from 9}

23. Invalid

	E, ~G
W	G, ~E

* 1 ~◇(G • E)
2 E
[∴ ~◇G
* 3 asm: ◇G
4 ∴ ☐~(G • E) {from 1}
5 W ∴ G {from 3}
* 6 ∴ ~(G • E) {from 4}
7 ∴ ~G {from 2 and 6}
* 8 W ∴ ~(G • E) {from 4}
9 W ∴ ~E {from 5 and 8}

24. The first premise is ambiguous. The box-inside form gives a valid argument (but with a false or questionable first premise); the box-outside form is invalid.

Valid

* 1 (R ⊃ ☐~W)
* 2 ◇W
[∴ ~R
3 asm: R
4 W ∴ W {from 2}
5 ∴ ☐~W {from 1 and 3}
6 W ∴ ~W {from 5}
7 ∴ ~R {from 3; 4 contradicts 6}

Invalid

	R, ~W
W	W, ~R

1 ☐(R ⊃ ~W)
* 2 ◇W
[∴ ~R
3 asm: R
4 W ∴ W {from 2}
* 5 ∴ (R ⊃ ~W) {from 1}
6 ∴ ~W {from 3 and 5}
* 7 W ∴ (R ⊃ ~W) {from 1}
8 W ∴ ~R {from 4 and 7}

26. Valid

1 □(~R ⊃ B)
* 2 ~◇B
[∴ □R
* 3 asm: ~□R
4 ∴ □~B {from 2}
* 5 ∴ ◇~R {from 3}
6 W ∴ ~R {from 5}
* 7 W ∴ (~R ⊃ B) {from 1}
8 W ∴ B {from 6 and 7}
9 W ∴ ~B {from 4}
10 ∴ □R {from 3; 8 contradicts 9}

27. The second premise is ambiguous. The box-inside form gives a valid argument (but with a false or questionable second premise); the box-outside form is invalid.

Valid

1 A
* 2 (A ⊃ □D)
* 3 (□D ⊃ ~F)
[∴ ~F
4 asm: F
5 ∴ □D {from 1 and 2}
6 ∴ ~□D {from 3 and 4}
7 ∴ ~F {from 4; 5 contradicts 6}

Invalid

1 A
2 □(A ⊃ D)
* 3 (□D ⊃ ~F)
[∴ ~F
4 asm: F
* 5 ∴ ~□D {from 3 and 4}
* 6 ∴ ◇~D {from 5}
7 W ∴ ~D {from 6}
* 8 ∴ (A ⊃ D) {from 2}
9 ∴ D {from 1 and 8}
* 10 W ∴ (A ⊃ D) {from 2}
11 W ∴ ~A {from 7 and 10}

	A, D, F
W	~A, ~D

8.1a

2. Valid in any system

* 1 ◇A
[∴ ◇◇A
* 2 asm: ~◇◇A
3 W ∴ A {from 1} # ⇒ W
4 ∴ □~◇A {from 2}
* 5 W ∴ ~◇A {from 4} any system
6 W ∴ □~A {from 5}
7 W ∴ ~A {from 6} any system
8 ∴ ◇◇A {from 2; 3 contradicts 7}

4. Valid in S5

* 1 ◇□A
[∴ □A
* 2 asm: ~□A
3 W ∴ □A {from 1} # ⇒ W
* 4 ∴ ◇~A {from 2}
5 WW ∴ ~A {from 4} # ⇒ WW
6 WW ∴ A {from 3} need S5
7 ∴ □A {from 2; 5 contradicts 6}

6. Valid in S4 or S5

1 □(A ⊃ B)
[∴ □(□A ⊃ □B)
* 2 asm: ~□(□A ⊃ □B)
* 3 ∴ ◇~(□A ⊃ □B) {from 2}
* 4 W ∴ ~(□A ⊃ □B) {from 3} # ⇒ W
5 W ∴ □A {from 4}
* 6 W ∴ ~□B {from 4}
* 7 W ∴ ◇~B {from 6}
8 WW ∴ ~B {from 7} W ⇒ WW
* 9 WW ∴ (A ⊃ B) {from 1} need S4 or S5
10 WW ∴ ~A {from 8 and 9}
11 WW ∴ A {from 5} any system
12 ∴ □(□A ⊃ □B) {from 2; 10 contradicts 11}

7. Valid in S4 or S5

* 1 (◇A ⊃ □B)
[∴ □(A ⊃ □B)
* 2 asm: ~□(A ⊃ □B)
* 3 ∴ ◇~(A ⊃ □B) {from 2}
* 4 W ∴ ~(A ⊃ □B) {from 3} # ⇒ W
5 W ∴ A {from 4}
* 6 W ∴ ~□B {from 4}
* 7 W ∴ ◇~B {from 6}
8 WW ∴ ~B {from 7} W ⇒ WW
9 asm: ~◇A {break up 1}
10 ∴ □~A {from 9}
11 W ∴ ~A {from 10} any system
12 ∴ ◇A {from 9; 5 contradicts 11}
13 ∴ □B {from 1 and 12}
14 WW ∴ B {from 13} need S4 or S5
15 ∴ □(A ⊃ □B) {from 2; 8 contradicts 14}

8. Valid in S5

1 □(A ⊃ □B)
[∴ (◇A ⊃ □B)

* 2 asm: ~(◇A ⊃ □B)
* 3 ∴ ◇A {from 2}
* 4 ∴ ~□B {from 2}
5 W ∴ A {from 3} # ⇒ W
* 6 ∴ ◇~B {from 4}
7 WW ∴ ~B {from 6} # ⇒ WW
* 8 W ∴ (A ⊃ □B) {from 1} any system
9 W ∴ □B {from 5 and 8}
10 WW ∴ B {from 9} need S5
11 ∴ (◇A ⊃ □B) {from 2; 7 contradicts 10}

9. Valid in S4 or S5

* 1 ◇□◇A
[∴ ◇A
* 2 asm: ~◇A
3 W ∴ □◇A {from 1} # ⇒ W
4 ∴ □~A {from 2}
* 5 W ∴ ◇A {from 3} any system
6 WW ∴ A {from 5} W ⇒ WW
7 WW ∴ ◇A {from 3} any system
8 W ∴ ~A {from 4} any system
9 WW ∴ ~A {from 4} need S4 or S5
10 ∴ ◇A {from 2; 6 contradicts 9}

11. Valid in S4 or S5

1 □A
[∴ □(B ⊃ □A)
* 2 asm: ~□(B ⊃ □A)
* 3 ∴ ◇~(B ⊃ □A) {from 2}
* 4 W ∴ ~(B ⊃ □A) {from 3} # ⇒ W
5 W ∴ B {from 4}
* 6 W ∴ ~□A {from 4}
* 7 W ∴ ◇~A {from 6}
8 WW ∴ ~A {from 7} W ⇒ WW
9 WW ∴ A {from 1} need S4 or S5
10 ∴ □(B ⊃ □A) {from 2; 8 contradicts 9}

12. Valid in B or S5

1 □◇□◇A
[∴ □◇A
* 2 asm: ~□◇A
* 3 ∴ ◇~◇A {from 2}
* 4 W ∴ ~◇A {from 3} # ⇒ W
5 W ∴ □~A {from 4}
* 6 W ∴ ◇□◇A {from 1} any system
7 WW ∴ □◇A {from 6} W ⇒ WW
8 W ∴ ◇A {from 7} need B or S5
9 ∴ □◇A {from 2; 4 contradicts 8}

13. Valid in S4 or S5

1 □◇A
[∴ □◇□◇A
* 2 asm: ~□◇□◇A
* 3 ∴ ◇~◇□◇A {from 2}
* 4 W ∴ ~◇□◇A {from 3} # ⇒ W
5 W ∴ □~□◇A {from 4}
* 6 W ∴ ~□◇A {from 5} any system
* 7 W ∴ ◇~◇A {from 6}
* 8 WW ∴ ~◇A {from 7} W ⇒ WW
9 WW ∴ ◇A {from 1} need S4 or S5
10 ∴ □◇□◇A {from 2; 8 contradicts 9}

14. Valid in S5

1 □(A ⊃ □B)
* 2 ◇A
[∴ □B
* 3 asm: ~□B
4 W ∴ A {from 2} # ⇒ W
* 5 ∴ ◇~B {from 3}
6 WW ∴ ~B {from 5} # ⇒ WW
* 7 W ∴ (A ⊃ □B) {from 1} any system
8 W ∴ □B {from 4 and 7}
9 WW ∴ B {from 8} need S5
10 ∴ □B {from 3; 6 contradicts 9}

8.1b

2. Valid in any system

* 1 ~(◇N • ◇~N)
* 2 ◇N
[∴ N
3 asm: ~N
* 4 ∴ ~◇~N {from 1 and 2}
5 ∴ □N {from 4}
6 ∴ N {from 5} any system
7 ∴ N {from 3; 3 contradicts 6}

4. Valid in any system

1 □(N ⊃ □N)
* 2 ◇~N
[∴ ~N
3 asm: N
4 W ∴ ~N {from 2} # ⇒ W
* 5 ∴ (N ⊃ □N) {from 1} any system
6 ∴ □N {from 3 and 5} any system
7 W ∴ N {from 6}
8 ∴ ~N {from 3; 4 contradicts 7}

8.2a

2. ◇(x)Ux

4. Ambiguous: (x)(Bx ⊃ □Ux) or □(x)(Bx ⊃ Ux)
6. Ambiguous: (x)(Mx ⊃ □Rx) or □(x)(Mx ⊃ Rx)
7. Ambiguous: (x)(Mx ⊃ (Tx • ◇~Tx)) or ((x)(Mx ⊃ Tx) • ◇~(x)(Mx ⊃ Tx))
8. □Sj
9. Ambiguous: (x)(Ox ⊃ □Sx) or □(x)(Ox ⊃ Sx)
11. □(x)(Lx ⊃ Px)
12. (x)(Lx ⊃ □Px)
13. (x)(Lx ⊃ (Px • ◇~Px))
14. (x)(Cx ⊃ ◇Tx)
16. □(x)x=x
17. (x)□x=x
18. Ambiguous: (x)((Mx • Tx) ⊃ □Tx) or □(x)((Mx • Tx) ⊃ Tx)
19. ◇□Ug

8.3a

2. Valid

  1 a=b
  [∴ (□Fa ⊃ □Fb)
* 2 asm: ~(□Fa ⊃ □Fb)
  3 ∴ □Fa {from 2}
  4 ∴ ~□Fb {from 2}
  5 ∴ ~□Fa {from 1 and 4}
  6 ∴ (□Fa ⊃ □Fb) {from 2; 3 contradicts 5}

4. Valid

  [∴ (∃x)□x=a
* 1 asm: ~(∃x)□x=a
  2 ∴ (x)~□x=a {from 1}
* 3 ∴ ~□a=a {from 2}
* 4 ∴ ◇~a=a {from 3}
  5 W ∴ ~a=a {from 4}
  6 W ∴ a=a {to contradict 5}
  7 ∴ (∃x)□x=a {from 1; 5 contradicts 6}

6. Valid

  [∴ (x)□x=x
* 1 asm: ~(x)□x=x
* 2 ∴ (∃x)~□x=x {from 1}
* 3 ∴ ~□a=a {from 2}
* 4 ∴ ◇~a=a {from 3}
  5 W ∴ ~a=a {from 4}
  6 W ∴ a=a {to contradict 5}
  7 ∴ (x)□x=x {from 1; 5 contradicts 6}

7. Valid

  [∴ □(x)x=x
* 1 asm: ~□(x)x=x
* 2 ∴ ◇~(x)x=x {from 1}
* 3 W ∴ ~(x)x=x {from 2}
* 4 W ∴ (∃x)~x=x {from 3}
  5 W ∴ ~a=a {from 4}
  6 W ∴ a=a {to contradict 5}
  7 ∴ □(x)x=x {from 1; 5 contradicts 6}

8. Invalid

  1 □(x)(Fx ⊃ Gx)
  [∴ (x)(Fx ⊃ □Gx)
* 2 asm: ~(x)(Fx ⊃ □Gx)
* 3 ∴ (∃x)~(Fx ⊃ □Gx) {from 2}
* 4 ∴ ~(Fa ⊃ □Ga) {from 3}
  5 ∴ Fa {from 4}
* 6 ∴ ~□Ga {from 4}
* 7 ∴ ◇~Ga {from 6}
  8 W ∴ ~Ga {from 7}
  9 ∴ (x)(Fx ⊃ Gx) {from 1}
  10 W ∴ (x)(Fx ⊃ Gx) {from 1}
* 11 ∴ (Fa ⊃ Ga) {from 9}
  12 ∴ Ga {from 5 and 11}
* 13 W ∴ (Fa ⊃ Ga) {from 10}
  14 W ∴ ~Fa {from 8 and 13}

	a
	Fa, Ga
W	~Fa, ~Ga

9. Valid

* 1 ◇(∃x)Fx
  [∴ (∃x)◇Fx
* 2 asm: ~(∃x)◇Fx
* 3 W ∴ (∃x)Fx {from 1}
  4 ∴ (x)~◇Fx {from 2}
  5 W ∴ Fa {from 3}
* 6 ∴ ~◇Fa {from 4}
  7 ∴ □~Fa {from 6}
  8 W ∴ ~Fa {from 7}
  9 ∴ (∃x)◇Fx {from 2; 5 contradicts 8}

11. Invalid

* 1 (◇(x)Fx ⊃ (x)◇Fx)
  [∴ ((∃x)~Fx ⊃ □(∃x)~Fx)
* 2 asm: ~((∃x)~Fx ⊃ □(∃x)~Fx)
* 3 ∴ (∃x)~Fx {from 2}
* 4 ∴ ~□(∃x)~Fx {from 2}
  5 ∴ ~Fa {from 3}
* 6 ∴ ◇~(∃x)~Fx {from 4}
* 7 W ∴ ~(∃x)~Fx {from 6}
  8 W ∴ (x)Fx {from 7}
  9 W ∴ Fa {from 8}

	a
	~Fa
W	Fa

10 ┌ asm: ~◇(x)Fx {break up 1}
11 │ ∴ ☐~(x)Fx {from 10}
12 │ ∴ ~(x)Fx {from 11}
13 │ ∴ (∃x)~Fx {from 12}
14 └ W ∴ ~(x)Fx {from 11}
15 ∴ ◇(x)Fx {from 10; 8 contradicts 14}
16 ∴ (x)◇Fx {from 1 and 15}
17 ∴ ◇Fa {from 16}

12. Valid

[∴ (x)(y)(x=y ⊃ ☐x=y)
* 1 ┌ asm: ~(x)(y)(x=y ⊃ ☐x=y)
* 2 │ ∴ (∃x)~(y)(x=y ⊃ ☐x=y) {from 1}
* 3 │ ∴ ~(y)(a=y ⊃ ☐a=y) {from 2}
* 4 │ ∴ (∃y)~(a=y ⊃ ☐a=y) {from 3}
* 5 │ ∴ ~(a=b ⊃ ☐a=b) {from 4}
6 │ ∴ a=b {from 5}
* 7 │ ∴ ~☐a=b {from 5}
* 8 │ ∴ ◇~a=b {from 7}
* 9 │ ∴ ◇~b=b {from 6 and 8}
10 │ W ∴ ~b=b {from 9}
11 └ W ∴ b=b {to contradict 10}
12 ∴ (x)(y)(x=y ⊃ ☐x=y) {from 1; 10 contradicts 11}

13. Valid

1 ☐(x)(Fx ⊃ Gx)
2 ☐Fa
[∴ ☐Ga
* 3 ┌ asm: ~☐Ga
* 4 │ ∴ ◇~Ga {from 3}
5 │ W ∴ ~Ga {from 4}
6 │ W ∴ (x)(Fx ⊃ Gx) {from 1}
7 │ W ∴ Fa {from 2}
* 8 │ W ∴ (Fa ⊃ Ga) {from 6}
9 └ W ∴ ~Fa {from 5 and 8}
10 ∴ ☐Ga {from 3; 7 contradicts 9}

14. Invalid

1 ~a=b
[∴ ☐~a=b
* 2 asm: ~☐~a=b
* 3 ∴ ◇a=b {from 2}
4 W ∴ a=b {from 3}

	a, b
	~a=b
W	a=b

8.3b

2. Valid

1 ☐(x)(~Bx ⊃ ~x=i)
[∴ ☐Bi
* 2 ┌ asm: ~☐Bi
* 3 │ ∴ ◇~Bi {from 2}
4 │ W ∴ ~Bi {from 3}
5 │ W ∴ (x)(~Bx ⊃ ~x=i) {from 1}
* 6 │ W ∴ (~Bi ⊃ ~i=i) {from 5}
7 │ W ∴ i=i {self-identity for 6}
8 └ W ∴ ~i=i {from 4 and 6}
9 ∴ ☐Bi {from 2; 7 contradicts 8}

4. The first premise is ambiguous. The box-inside form gives a valid argument; the box-outside form is invalid.

Valid

1 (x)(Mx ⊃ ☐Rx)
2 Mp
[∴ ☐Rp
* 3 ┌ asm: ~☐Rp
* 4 │ ∴ ◇~Rp {from 3}
* 5 │ ∴ (Mp ⊃ ☐Rp) {from 1}
6 └ ∴ ☐Rp {from 2 and 5}
7 ∴ ☐Rp {from 3; 3 contradicts 6}

Invalid

1 ☐(x)(Mx ⊃ Rx)
2 Mp
[∴ ☐Rp
* 3 asm: ~☐Rp
* 4 ∴ ◇~Rp {from 3}
5 W ∴ ~Rp {from 4}
6 ∴ (x)(Mx ⊃ Rx) {from 1}
7 W ∴ (x)(Mx ⊃ Rx) {from 1}
* 8 ∴ (Mp ⊃ Rp) {from 6}
9 ∴ Rp {from 2 and 8}
* 10 W ∴ (Mp ⊃ Rp) {from 7}
11 W ∴ ~Mp {from 5 and 10}

	p
	Mp, Rp
W	~Mp, ~Rp

6. Valid (but see Section 8.4)

* 1 ~☐En
2 e=n
[∴ ~☐Ee
3 ┌ asm: ☐Ee
4 └ ∴ ☐En {from 2 and 3}
5 ∴ ~☐Ee {from 3; 1 contradicts 4}

7. Valid

* 1 ◇(Ti • ~(∃x)Mx)
2 (x)(Mx ⊃ ☐Mx)
[∴ ~Mi
3 ┌ asm: Mi
* 4 │ W ∴ (Ti • ~(∃x)Mx) {from 1}

5 W ∴ Ti {from 4}
* 6 W ∴ ~(∃x)Mx {from 4}
7 W ∴ (x)~Mx {from 6}
* 8 ∴ (Mi ⊃ ☐Mi) {from 2}
9 ∴ ☐Mi {from 3 and 8}
10 W ∴ ~Mi {from 7}
11 W ∴ Mi {from 9}
12 ∴ ~Mi {from 3; 10 contradicts 11}

8. The first premise and conclusion are ambiguous. It's valid if we take both as box-inside forms; it's invalid if we take both as box-outside forms (or if we take one as box-inside and the other as box-outside).

Valid

1 (x)(Hx ⊃ ☐Rx)
2 (x)(Lx ⊃ Hx)
[∴ (x)(Lx ⊃ ☐Rx)
* 3 asm: ~(x)(Lx ⊃ ☐Rx)
* 4 ∴ (∃x)~(Lx ⊃ ☐Rx) {from 3}
* 5 ∴ ~(La ⊃ ☐Ra) {from 4}
6 ∴ La {from 5}
* 7 ∴ ~☐Ra {from 5}
* 8 ∴ ◇~Ra {from 7}
9 W ∴ ~Ra {from 8}
*10 ∴ (Ha ⊃ ☐Ra) {from 1}
11 ∴ ~Ha {from 7 and 10}
*12 ∴ (La ⊃ Ha) {from 2}
13 ∴ Ha {from 6 and 12}
14 ∴ (x)(Lx ⊃ ☐Rx) {from 3; 11 contradicts 13}

Invalid

	a
	~La, ~Ha
W	La, ~Ra, ~Ha

1 ☐(x)(Hx ⊃ Rx)
2 (x)(Lx ⊃ Hx)
[∴ ☐(x)(Lx ⊃ Rx)
* 3 asm: ~☐(x)(Lx ⊃ Rx)
* 4 ∴ ◇~(x)(Lx ⊃ Rx) {from 3}
* 5 W ∴ ~(x)(Lx ⊃ Rx) {from 4}
* 6 W ∴ (∃x)~(Lx ⊃ Rx) {from 5}
* 7 W ∴ ~(La ⊃ Ra) {from 6}
8 W ∴ La {from 7}
9 W ∴ ~Ra {from 7}
10 ∴ (x)(Hx ⊃ Rx) {from 1}
11 W ∴ (x)(Hx ⊃ Rx) {from 1}
12 ∴ (La ⊃ Ha) {from 2}
13 ∴ (Ha ⊃ Ra) {from 10}
*14 W ∴ (Ha ⊃ Ra) {from 11}
15 W ∴ ~Ha {from 9 and 14}
16 asm: ~La {break up 12}
17 asm: ~Ha {break up 13}

9. Invalid

	a, p
	Cp, Rp
W	Ca, ~Ra, Rp

* 1 ~☐(x)(Cx ⊃ Rx)
2 Cp
3 Rp
[∴ (Rp • ◇~Rp)
* 4 asm: ~(Rp • ◇~Rp)
* 5 ∴ ◇~(x)(Cx ⊃ Rx) {from 1}
* 6 W ∴ ~(x)(Cx ⊃ Rx) {from 5}
* 7 W ∴ (∃x)~(Cx ⊃ Rx) {from 6}
* 8 W ∴ ~(Ca ⊃ Ra) {from 7}
9 W ∴ Ca {from 8}
10 W ∴ ~Ra {from 8}
*11 ∴ ~◇~Rp {from 3 and 4}
12 ∴ ☐Rp {from 11}
13 W ∴ Rp {from 12}

11. Valid

* 1 (M ⊃ ◇(∃x)Ax)
2 ☐(x)(Ax ⊃ Rx)
3 ☐~(∃x)(Rx • Ax)
[∴ ~M
4 asm: M
* 5 ∴ ◇(∃x)Ax {from 1 and 4}
* 6 W ∴ (∃x)Ax {from 5}
7 W ∴ Aa {from 6}
8 W ∴ (x)(Ax ⊃ Rx) {from 2}
* 9 W ∴ ~(∃x)(Rx • Ax) {from 3}
10 W ∴ (x)~(Rx • Ax) {from 9}
*11 W ∴ (Aa ⊃ Ra) {from 8}
12 W ∴ Ra {from 7 and 11}
*13 W ∴ ~(Ra • Aa) {from 10}
14 W ∴ ~Ra {from 7 and 13}
25 ∴ ~M {from 4; 12 contradicts 14}

12. Valid (but see Section 8.4)

1 n=t
2 ☐Gte
[∴ ☐Gne
3 asm: ~☐Gne
4 ∴ ~☐Gte {from 1 and 3}
5 ∴ ☐Gne {from 3; 2 contradicts 4}

13. The first premise is ambiguous. The box-inside form gives a valid argument (but has a false first premise); the box-outside form is invalid.

Valid

1 (x)(Cx ⊃ ☐Tx)

2 Cp
[∴ ☐Tp
* 3 asm: ~☐Tp
* 4 ∴ (Cp ⊃ ☐Tp) {from 1}
5 ∴ ☐Tp {from 2 and 4}
6 ∴ ☐Tp {from 3; 3 contradicts 5}

Invalid

1 ☐(x)(Cx ⊃ Tx)
2 Cp
[∴ ☐Tp
* 3 asm: ~☐Tp
* 4 ∴ ◇~Tp {from 3}
5 W ∴ ~Tp {from 4}
6 ∴ (x)(Cx ⊃ Tx) {from 1}
7 W ∴ (x)(Cx ⊃ Tx) {from 1}
* 8 ∴ (Cp ⊃ Tp) {from 6}
9 ∴ Tp {from 2 and 8}
* 10 W ∴ (Cp ⊃ Tp) {from 7}
11 W ∴ ~Cp {from 5 and 10}

	p
	Cp, Tp
W	~Cp, ~Tp

14. Valid

* 1 (∃x)(Ux • ~◇(∃y)Gyx)
2 (x)(y)((Rx • ~Ry) ⊃ Gxy)
3 Rs
[∴ (∃x)(Rx • ~◇(∃y)Gyx)
* 4 asm: ~(∃x)(Rx • ~◇(∃y)Gyx)
5 ∴ (x)~(Rx • ~◇(∃y)Gyx) {from 4}
* 6 ∴ (Ua • ~◇(∃y)Gya) {from 1}
7 ∴ Ua {from 6}
* 8 ∴ ~◇(∃y)Gya {from 6}
* 9 ∴ ~(Ra • ~◇(∃y)Gya) {from 5}
10 ∴ ~Ra {from 5}
11 ∴ (y)((Rs • ~Ry) ⊃ Gsy) {from 2}
12 ∴ ((Rs • ~Ra) ⊃ Gsa) {from 11}
13 ∴ ☐~(∃y)Gya {from 8}
14 ∴ ~(∃y)Gya {from 13}
15 ∴ (y)~Gya {from 14}
16 ∴ ~Gsa {from 15}
17 ∴ ~(Rs • ~Ra) {from 12 and 16}
18 ∴ ~Rs {from 10 and 17}
19 ∴ ~☐(x)(Px ⊃ Bx) {from 4; 3 contradicts 18}

9.1a

2. (~L ⊃ $\underline{S}$)
4. (A$\underline{u}$ ⊃ Wu) *or, equivalently,* (~Wu ⊃ ~A$\underline{u}$)
6. (A ⊃ ~$\underline{B}$)
7. (B ⊃ ~$\underline{A}$)
8. (B ⊃ $\underline{A}$)
9. ~($\underline{B}$ • ~$\underline{A}$)
11. (Cu ⊃ D$\underline{u}$)
12. (x)(Cx ⊃ D$\underline{x}$)
13. Rg$\underline{j}$
14. (Hju ⊃ H$\underline{u}$j)
16. (A ⊃ ~$\underline{B}$)
17. ~($\underline{B}$ • $\underline{A}$)
18. (∃x)(S$\underline{x}$ • W$\underline{x}$)
19. (∃x)(Sx • W$\underline{x}$)
21. (∃x)(Lux • W$\underline{u}$x)
22. (∃x)(Wux • L$\underline{u}$x)

9.2a

2. Invalid

A, ~$\underline{A}$, ~$\underline{B}$

* 1 ~($\underline{A}$ • ~$\underline{B}$)
[∴ (A ⊃ $\underline{B}$)
* 2 asm: ~(A ⊃ $\underline{B}$)
3 ∴ A {from 2}
4 ∴ ~$\underline{B}$ {from 2}
5 ∴ ~$\underline{A}$ {from 1 and 4}

4. Invalid

$\underline{A}$, ~$\underline{B}$, ~A

* 1 (A ⊃ $\underline{B}$)
[∴ ~($\underline{A}$ • ~$\underline{B}$)
* 2 asm: ($\underline{A}$ • ~$\underline{B}$)
3 ∴ $\underline{A}$ {from 2}
4 ∴ ~$\underline{B}$ {from 2}
5 ∴ ~A {from 1 and 4}

6. Invalid

a
F$\underline{a}$, ~G$\underline{a}$, ~Fa

1 (x)(Fx ⊃ G$\underline{x}$)
2 F$\underline{a}$
[∴ G$\underline{a}$
3 asm: ~G$\underline{a}$
* 4 ∴ (Fa ⊃ G$\underline{a}$) {from 1}
5 ∴ ~Fa {from 3 and 4}

7. Valid

1 (x)~(F$\underline{x}$ • G$\underline{x}$)
2 (x)(Hx ⊃ F$\underline{x}$)
[∴ (x)(G$\underline{x}$ ⊃ ~Hx)
* 3 asm: ~(x)(G$\underline{x}$ ⊃ ~Hx)
* 4 ∴ (∃x)~(G$\underline{x}$ ⊃ ~Hx) {from 3}
* 5 ∴ ~(G$\underline{a}$ ⊃ ~Ha) {from 4}
6 ∴ G$\underline{a}$ {from 5}
7 ∴ Ha {from 5}
* 8 ∴ ~(F$\underline{a}$ • G$\underline{a}$) {from 1}
9 ∴ ~F$\underline{a}$ {from 6 and 8}

*10 ∴ (Ha ⊃ Fa) {from 2}
11 ∴ Fa {from 7 and 10}
12 ∴ (x)(Gx ⊃ ~Hx) {from 3; 9 contradicts 11}

8. Invalid

a

Fa, ~Ha
Ga, ~Ga

1 (x)(Fx ⊃ Gx)
2 (x)(Gx ⊃ Hx)
[∴ (x)(Fx ⊃ Hx)
* 3 asm: ~(x)(Fx ⊃ Hx)
* 4 ∴ (∃x)~(Fx ⊃ Hx) {from 3}
* 5 ∴ ~(Fa ⊃ Ha) {from 4}
6 ∴ Fa {from 5}
7 ∴ ~Ha {from 5}
* 8 ∴ (Fa ⊃ Ga) {from 1}
9 ∴ Ga {from 6 and 8}
*10 ∴ (Ga ⊃ Ha) {from 2}
11 ∴ ~Ga {from 7 and 10}

9. Valid

* 1 (~A ∨ ~B)
[∴ ~(A • B)
* 2 asm: (A • B)
3 ∴ A {from 2}
4 ∴ B {from 2}
5 ∴ ~B {from 1 and 3}
6 ∴ ~(A • B) {from 2; 4 contradicts 5}

9.2b

2. Invalid

~E, ~G, E

1 ~E
* 2 (~E ⊃ G)
[∴ G
3 asm: ~G
4 ∴ E {from 2 and 3}

4. Valid

* 1 (S ⊃ P)
2 S
[∴ P
3 asm: ~P
4 ∴ P {from 1 and 2}
5 ∴ P {from 3; 3 contradicts 4}

6. Valid

* 1 (B ⊃ C)
2 B
[∴ C
3 asm: ~C
4 ∴ C {from 1 and 2}
5 ∴ C {from 3; 3 contradicts 4}

7. Invalid

B, ~C, ~B

* 1 ~(B • ~C)
2 B
[∴ C
3 asm: ~C
4 ∴ ~B {from 1 and 3}

8. Valid

* 1 ~(E • ~M)
2 ~M
[∴ ~E
3 asm: E
4 ∴ ~E {from 1 and 2}
5 ∴ ~E {from 3; 3 contradicts 4}

9. Valid

* 1 (L ⊃ W)
2 ~W
[∴ ~L
3 asm: L
4 ∴ ~L {from 1 and 2}
5 ∴ ~L {from 3; 3 contradicts 4}

11. Valid

1 w=l
[∴ (Guw ∨ ~Gul)
* 2 asm: ~(Guw ∨ ~Gul)
3 ∴ ~Guw {from 2}
4 ∴ ~Gul {from 1 and 3}
5 ∴ Gul {from 2}
6 ∴ (Guw ∨ ~Gul) {from 2; 4 contradicts 5}

12. Valid

* 1 (∃x)(Ax • Dux)
2 (x)(Ax ⊃ (Jx ∨ Sx))
[∴ (∃x)((Jx ∨ Sx) • Dux)
* 3 asm: ~(∃x)((Jx ∨ Sx) • Dux)
* 4 ∴ (Aa • Dua) {from 1}
5 ∴ (x)~((Jx ∨ Sx) • Dux) {from 3}
6 ∴ Aa {from 4}
7 ∴ Dua {from 4}
* 8 ∴ (Aa ⊃ (Ja ∨ Sa)) {from 2}
9 ∴ (Ja ∨ Sa) {from 6 and 8}
10 ∴ (Au ⊃ (Ju ∨ Su)) {from 2}
*11 ∴ ~((Ja ∨ Sa) • Dua) {from 5}
12 ∴ ~(Ja ∨ Sa) {from 7 and 11}
13 ∴ (∃x)((Jx ∨ Sx) • Dux) {from 3; 9 contradicts 12}

13. Valid

* 1 $(B \supset \underline{R})$
* 2 $(S \supset B)$
[$\therefore (S \supset \underline{R})$
* 3 asm: $\sim(S \supset \underline{R})$
4 $\therefore S$ {from 3}
5 $\therefore \sim\underline{R}$ {from 3}
6 $\therefore \sim B$ {from 1 and 5}
7 $\therefore B$ {from 2 and 4}
8 $\therefore (S \supset \underline{R})$ {from 3; 6 contradicts 7}

14. Valid

1 $\sim\underline{S}$
[$\therefore \sim(\underline{S} \cdot \sim\underline{P})$
2 asm: $(\underline{S} \cdot \sim\underline{P})$
3 $\therefore \underline{S}$ {from 2}
4 $\therefore \sim(\underline{S} \cdot \sim\underline{P})$ {from 2; 1 contradicts 3}

16. Valid

1 $(x)(Dx \supset B\underline{u}x)$
2 Dt
[$\therefore B\underline{u}t$
3 asm: $\sim B\underline{u}t$
* 4 $\therefore (Dt \supset B\underline{u}t)$ {from 1}
5 $\therefore B\underline{u}t$ {from 2 and 4}
6 $\therefore B\underline{u}t$ {from 3; 3 contradicts 5}

17. Invalid $\boxed{\underline{T}, \sim\underline{M}, \sim T}$

1 $(T \supset M)$
2 $\underline{T}$
[$\therefore \underline{M}$
3 asm: $\sim\underline{M}$
4 asm: $\sim T$ {break up 1}

18. Invalid $\boxed{\underline{G}, \sim\underline{P}, \sim G}$

1 $\underline{G}$
* 2 $(G \supset \underline{P})$
[$\therefore \underline{P}$
3 asm: $\sim\underline{P}$
4 $\therefore \sim G$ {from 2 and 3}

19. Valid

[$\therefore (\underline{A} \vee \sim\underline{A})$
* 1 asm: $\sim(\underline{A} \vee \sim\underline{A})$
2 $\therefore \sim\underline{A}$ {from 1}
3 $\therefore \underline{A}$ {from 1}
4 $\therefore (\underline{A} \vee \sim\underline{A})$ {from 1; 2 contradicts 3}

21. Valid

1 $\underline{M}$
[$\therefore (\underline{M} \vee \underline{B})$
2 asm: $\sim(\underline{M} \vee \underline{B})$
3 $\therefore \sim\underline{M}$ {from 2}
4 $\therefore (\underline{M} \vee \underline{B})$ {from 2; 1 contradicts 3}

22. Invalid

a

$\boxed{\sim Ea, H\underline{a}, \sim Ha}$

1 $(x)(Hx \supset E\underline{x})$
[$\therefore (x)(\sim Ex \supset \sim H\underline{x})$
* 2 asm: $\sim(x)(\sim Ex \supset \sim H\underline{x})$
* 3 $\therefore (\exists x)\sim(\sim Ex \supset \sim H\underline{x})$ {from 2}
* 4 $\therefore \sim(\sim Ea \supset \sim H\underline{a})$ {from 3}
5 $\therefore \sim Ea$ {from 4}
6 $\therefore H\underline{a}$ {from 4}
7 $\therefore (Ha \supset E\underline{a})$ {from 1}
8 asm: $\sim Ha$ {break up 7}

9.3a

2. $O\sim(\underline{A} \cdot \underline{B})$
4. $(\underline{A} \supset R\underline{A})$
6. $(R\underline{A} \cdot R\sim\underline{A})$
7. $((R\underline{A} \cdot R\underline{B}) \supset R(\underline{A} \cdot \underline{B}))$
8. $(\sim O\underline{A} \cdot O\sim\underline{A})$
9. $(B \supset O\underline{A})$
11. $\sim\Box((x)Ax \supset RA\underline{u})$
12. $(RA\underline{x}y \supset RA\underline{y}x)$
13. $(O\underline{A} \supset \Diamond A)$
14. $O(x)(S\underline{s}x \supset Gx)$
16. $(R(\exists x)A\underline{x} \supset R(x)A\underline{x})$
17. $(RA\underline{u} \supset (x)RA\underline{x})$
18. $(x)\sim RA\underline{x}$ *or, equivalently,* $\sim R(\exists x)A\underline{x}$
19. $R(x)(\sim Sx \supset T\underline{x})$

9.4a

2. Valid

* 1 $(\exists x)OA\underline{x}$
[$\therefore O(\exists x)A\underline{x}$
* 2 asm: $\sim O(\exists x)A\underline{x}$
3 $\therefore OA\underline{a}$ {from 1}
* 4 $\therefore R\sim(\exists x)A\underline{x}$ {from 2}
* 5 D $\therefore \sim(\exists x)A\underline{x}$ {from 4}
6 D $\therefore (x)\sim A\underline{x}$ {from 5}
7 D $\therefore A\underline{a}$ {from 3}
8 D $\therefore \sim A\underline{a}$ {from 6}
9 $\therefore O(\exists x)A\underline{x}$ {from 2; 7 contradicts 8}

4. Valid

[$\therefore O(O\underline{A} \supset \underline{A})$
* 1 asm: $\sim O(O\underline{A} \supset \underline{A})$
* 2 $\therefore R\sim(O\underline{A} \supset \underline{A})$ {from 1}
* 3 D $\therefore \sim(O\underline{A} \supset \underline{A})$ {from 2}

4 $D \therefore O\underline{A}$ {from 3}
5 $D \therefore \sim\underline{A}$ {from 3}
6 $D \therefore \underline{A}$ {from 4}
7 $\therefore O(O\underline{A} \supset \underline{A})$ {from 1; 5 contradicts 6}

6. Valid

[$\therefore O(\underline{A} \supset R\underline{A})$
* 1 asm: $\sim O(\underline{A} \supset R\underline{A})$
* 2 $\therefore R\sim(\underline{A} \supset R\underline{A})$ {from 1}
* 3 $D \therefore \sim(\underline{A} \supset R\underline{A})$ {from 2}
4 $D \therefore \underline{A}$ {from 3}
* 5 $D \therefore \sim R\underline{A}$ {from 3}
6 $D \therefore O\sim\underline{A}$ {from 5}
7 $D \therefore \sim\underline{A}$ {from 6}
8 $\therefore O(\underline{A} \supset R\underline{A})$ {from 1; 4 contradicts 7}

7. Valid

1 $O\underline{A}$
2 $O\underline{B}$
[$\therefore O(\underline{A} \cdot \underline{B})$
* 3 asm: $\sim O(\underline{A} \cdot \underline{B})$
* 4 $\therefore R\sim(\underline{A} \cdot \underline{B})$ {from 3}
* 5 $D \therefore \sim(\underline{A} \cdot \underline{B})$ {from 4}
6 $D \therefore \underline{A}$ {from 1}
7 $D \therefore \sim\underline{B}$ {from 5 and 6}
8 $D \therefore \underline{B}$ {from 2}
9 $\therefore O(\underline{A} \cdot \underline{B})$ {from 3; 7 contradicts 8}

8. Valid

1 $(x)OF\underline{x}$
[$\therefore O(x)F\underline{x}$
* 2 asm: $\sim O(x)F\underline{x}$
* 3 $\therefore R\sim(x)F\underline{x}$ {from 2}
* 4 $D \therefore \sim(x)F\underline{x}$ {from 3}
* 5 $D \therefore (\exists x)\sim F\underline{x}$ {from 4}
6 $D \therefore \sim F\underline{a}$ {from 5}
7 $\therefore OF\underline{a}$ {from 1}
8 $D \therefore F\underline{a}$ {from 7}
9 $\therefore O(x)F\underline{x}$ {from 2; 6 contradicts 8}

9. Valid

1 $O(\underline{A} \lor \underline{B})$
[$\therefore (\sim\diamond A \supset R\underline{B})$
* 2 asm: $\sim(\sim\diamond A \supset R\underline{B})$
* 3 $\therefore \sim\diamond A$ {from 2}
* 4 $\therefore \sim R\underline{B}$ {from 2}
5 $\therefore O\sim\underline{B}$ {from 4}
* 6 asm: $\sim O\underline{A}$ {nice to have "O$\underline{A}$" to use Kant's Law on to contradict 3}
7 $\therefore R\sim\underline{A}$ {from 6}
8 $D \therefore \sim\underline{A}$ {from 7}
9 $D \therefore (\underline{A} \lor \underline{B})$ {from 1}
10 $D \therefore \underline{B}$ {from 8 and 9}
11 $D \therefore \sim\underline{B}$ {from 5}
12 $\therefore O\underline{A}$ {from 6; 10 contradicts 11}
13 $\therefore \diamond A$ {from 12 using Kant's Law}
14 $\therefore (\sim\diamond A \supset R\underline{B})$ {from 2; 3 contradicts 13}

11. Valid

1 $\Box(\underline{A} \supset \underline{B})$
2 $O\underline{A}$
[$\therefore O\underline{B}$
* 3 asm: $\sim O\underline{B}$
* 4 $\therefore R\sim\underline{B}$ {from 3}
5 $D \therefore \sim\underline{B}$ {from 4}
* 6 $D \therefore (\underline{A} \supset \underline{B})$ {from 1}
7 $D \therefore \sim\underline{A}$ {from 5 and 6}
8 $D \therefore \underline{A}$ {from 2}
9 $\therefore O\underline{B}$ {from 3; 7 contradicts 8}

12. Valid

1 $O\underline{A}$
* 2 $R\underline{B}$
[$\therefore R(\underline{A} \cdot \underline{B})$
* 3 asm: $\sim R(\underline{A} \cdot \underline{B})$
4 $D \therefore \underline{B}$ {from 2}
5 $\therefore O\sim(\underline{A} \cdot \underline{B})$ {from 3}
6 $D \therefore \underline{A}$ {from 1}
* 7 $D \therefore \sim(\underline{A} \cdot \underline{B})$ {from 5}
8 $D \therefore \sim\underline{A}$ {from 4 and 7}
9 $\therefore R(\underline{A} \cdot \underline{B})$ {from 3; 6 contradicts 8}

13. Valid

1 A
[$\therefore O(\underline{B} \lor \sim\underline{B})$
* 2 asm: $\sim O(\underline{B} \lor \sim\underline{B})$
* 3 $\therefore R\sim(\underline{B} \lor \sim\underline{B})$ {from 2}
* 4 $D \therefore \sim(\underline{B} \lor \sim\underline{B})$ {from 3}
5 $D \therefore \sim\underline{B}$ {from 4}
6 $D \therefore \underline{B}$ {from 4}
7 $\therefore O(\underline{B} \lor \sim\underline{B})$ {from 2; 5 contradicts 6}

14. Invalid

1 $(x)RA\underline{x}$
[$\therefore R(x)A\underline{x}$
* 2 asm: $\sim R(x)A\underline{x}$
3 $\therefore O\sim(x)A\underline{x}$ {from 2}
* 4 $\therefore RA\underline{a}$ {from 1}
5 $D \therefore A\underline{a}$ {from 4}
* 6 $D \therefore \sim(x)A\underline{x}$ {from 3}
7 $D \therefore (\exists x)\sim A\underline{x}$ {from 6}
8 $D \therefore \sim A\underline{b}$ {from 7}

	a, b
D	$A\underline{a}, \sim A\underline{b}$
DD	$A\underline{b}, \sim A\underline{a}$

* 9 ∴ RA$\underline{b}$ {from 1}
10 DD ∴ A$\underline{b}$ {from 9}

Endless loop: add "~A$\underline{a}$" to world DD to make the conclusion false. (Refutations aren't required in this exercise.)

16. Valid

[∴ (R$\underline{A}$ ∨ R~$\underline{A}$)
* 1 ┌ asm: ~(R$\underline{A}$ ∨ R~$\underline{A}$)
* 2 │ ∴ ~R$\underline{A}$ {from 1}
* 3 │ ∴ ~R~$\underline{A}$ {from 1}
4 │ ∴ O~$\underline{A}$ {from 2}
5 │ ∴ O$\underline{A}$ {from 3}
6 │ ∴ ~$\underline{A}$ {from 4}
7 └ ∴ $\underline{A}$ {from 5}
8 ∴ (R$\underline{A}$ ∨ R~$\underline{A}$) {from 1; 6 contradicts 7}

17. Invalid

1 (O$\underline{A}$ ⊃ $\underline{B}$)
[∴ R($\underline{A}$ • $\underline{B}$)
* 2 asm: ~R($\underline{A}$ • $\underline{B}$)
3 ∴ O~($\underline{A}$ • $\underline{B}$) {from 2}
**4 asm: ~O$\underline{A}$ {break up 1}
**5 ∴ R~$\underline{A}$ {from 4}
6 D ∴ ~$\underline{A}$ {from 5}
7 D ∴ ~($\underline{A}$ • $\underline{B}$) {from 3}

18. Valid

* 1 ~◇A
[∴ R~$\underline{A}$
* 2 ┌ asm: ~R~$\underline{A}$
3 │ ∴ □~A {from 1}
4 │ ∴ O$\underline{A}$ {from 2}
5 │ ∴ ~A {from 3}
6 └ ∴ ◇A {from 4 by Kant's Law}
7 ∴ R~$\underline{A}$ {from 2; 1 contradicts 6}

19. Valid

1 A
2 ~A
[∴ O$\underline{B}$
3 [asm: ~O$\underline{B}$
4 ∴ O$\underline{B}$ {from 3; 1 contradicts 2}

21. Valid

1 O(A ⊃ $\underline{B}$)
[∴ (A ⊃ O$\underline{B}$)
* 2 ┌ asm: ~(A ⊃ O$\underline{B}$)
3 │ ∴ A {from 2}
* 4 │ ∴ ~O$\underline{B}$ {from 2}
* 5 │ ∴ R~$\underline{B}$ {from 4}
6 │ D ∴ ~$\underline{B}$ {from 5}
* 7 │ D ∴ (A ⊃ $\underline{B}$) {from 1}
8 │ D ∴ ~A {from 6 and 7}
9 └ ∴ ~A {from 8 by indicative transfer}
10 ∴ (A ⊃ O$\underline{B}$) {from 2; 3 contradicts 9}

22. Valid

1 O(x)A$\underline{x}$
[∴ (x)OA$\underline{x}$
* 2 ┌ asm: ~(x)OA$\underline{x}$
* 3 │ ∴ (∃x)~OA$\underline{x}$ {from 2}
* 4 │ ∴ ~OA$\underline{a}$ {from 3}
* 5 │ ∴ R~A$\underline{a}$ {from 4}
6 │ D ∴ ~A$\underline{a}$ {from 5}
7 │ D ∴ (x)A$\underline{x}$ {from 1}
8 └ D ∴ A$\underline{a}$ {from 7}
9 ∴ (x)OA$\underline{x}$ {from 2; 6 contradicts 8}

23. Valid

[∴ O(~R$\underline{A}$ ⊃ ~$\underline{A}$)
* 1 ┌ asm: ~O(~R$\underline{A}$ ⊃ ~$\underline{A}$)
* 2 │ ∴ R~(~R$\underline{A}$ ⊃ ~$\underline{A}$) {from 1}
* 3 │ D ∴ ~(~R$\underline{A}$ ⊃ ~$\underline{A}$) {from 2}
* 4 │ D ∴ ~R$\underline{A}$ {from 3}
5 │ D ∴ $\underline{A}$ {from 3}
6 │ D ∴ O~$\underline{A}$ {from 4}
7 └ D ∴ ~$\underline{A}$ {from 6}
8 ∴ O(~R$\underline{A}$ ⊃ ~$\underline{A}$) {from 1; 5 contradicts 7}

24. Valid

1 A
[∴ (A ∨ O$\underline{B}$)
2 ┌ asm: ~(A ∨ O$\underline{B}$)
3 └ ∴ ~A {from 2}
4 ∴ (A ∨ O$\underline{B}$) {from 2; 1 contradicts 3}

9.4b

2. Invalid

[∴ (O$\underline{A}$ ∨ O~$\underline{A}$)
* 1 asm: ~(O$\underline{A}$ ∨ O~$\underline{A}$)
* 2 ∴ ~O$\underline{A}$ {from 1}
* 3 ∴ ~O~$\underline{A}$ {from 1}
* 4 ∴ R~$\underline{A}$ {from 2}
* 5 ∴ R$\underline{A}$ {from 3}
6 D ∴ ~$\underline{A}$ {from 4}
7 DD ∴ $\underline{A}$ {from 5}

4. Valid

[∴ (O$\underline{A}$ ⊃ $\underline{A}$)
* 1 ┌ asm: ~(O$\underline{A}$ ⊃ $\underline{A}$)

2 ∴ O$\underline{A}$ {from 1}
3 ∴ ~$\underline{A}$ {from 1}
4 ∴ $\underline{A}$ {from 2}
5 ∴ (O$\underline{A}$ ⊃ $\underline{A}$) {from 1; 3 contradicts 4}

6. Valid

* 1 ~◇A
[∴ ~O$\underline{A}$
2 asm: O$\underline{A}$
3 ∴ □~A {from 1}
4 ∴ ◇A {from 2 by Kant's Law}
5 ∴ ~O$\underline{A}$ {from 2; 1 contradicts 4}

7. Invalid

1 O~($\underline{K}$ • $\underline{V}$)
* 2 ~O$\underline{V}$
[∴ R$\underline{K}$
* 3 asm: ~R$\underline{K}$
* 4 ∴ R~$\underline{V}$ {from 2}
5 ∴ O~$\underline{K}$ {from 3}
6 D ∴ ~$\underline{V}$ {from 4}
7 D ∴ ~($\underline{K}$ • $\underline{V}$) {from 1}
8 D ∴ ~$\underline{K}$ {from 5}

8. Valid

[∴ ($\underline{A}$ ⊃ R$\underline{A}$)
* 1 asm: ~($\underline{A}$ ⊃ R$\underline{A}$)
2 ∴ $\underline{A}$ {from 1}
* 3 ∴ ~R$\underline{A}$ {from 1}
4 ∴ O~$\underline{A}$ {from 3}
5 ∴ ~$\underline{A}$ {from 4}
6 ∴ ($\underline{A}$ ⊃ R$\underline{A}$) {from 1; 2 contradicts 5}

9. Valid

* 1 (RI$\underline{u}$j ⊃ RI$\underline{j}$u)
[∴ (O~I$\underline{j}$u ⊃ ~I$\underline{u}$j)
* 2 asm: ~(O~I$\underline{j}$u ⊃ ~I$\underline{u}$j)
3 ∴ O~I$\underline{j}$u {from 2}
4 ∴ I$\underline{u}$j {from 2}
5 ∴ ~I$\underline{j}$u {from 3}
6 asm: ~RI$\underline{u}$j {break up 1}
7 ∴ O~I$\underline{u}$j {from 6}
8 ∴ ~I$\underline{u}$j {from 7}
9 ∴ RI$\underline{u}$j {from 6; 4 contradicts 8}
* 10 ∴ RI$\underline{j}$u {from 1 and 9}
11 D ∴ I$\underline{j}$u {from 10}
12 D ∴ ~I$\underline{j}$u {from 3}
13 ∴ (O~I$\underline{j}$u ⊃ ~I$\underline{u}$j) {from 2; 11 contradicts 12}

11. Valid

* 1 ((F • A) ⊃ □A)
[∴ ((F • A) ⊃ R$\underline{A}$)
* 2 asm: ~((F • A) ⊃ R$\underline{A}$)
* 3 ∴ (F • A) {from 2}
* 4 ∴ ~R$\underline{A}$ {from 2}
5 ∴ F {from 3}
6 ∴ A {from 3}
7 ∴ O~$\underline{A}$ {from 4}
8 ∴ □A {from 1 and 3}
* 9 ∴ ◇~A {from 7 by Kant's Law}
10 W ∴ ~A {from 9}
11 W ∴ A {from 8}
12 ∴ ((F • A) ⊃ R$\underline{A}$) {from 2; 10 contradicts 11}

12. Valid

* 1 (R$\underline{C}$ ⊃ O$\underline{T}$)
[∴ O($\underline{T}$ ∨ ~$\underline{C}$)
* 2 asm: ~O($\underline{T}$ ∨ ~$\underline{C}$)
* 3 ∴ R~($\underline{T}$ ∨ ~$\underline{C}$) {from 2}
* 4 D ∴ ~($\underline{T}$ ∨ ~$\underline{C}$) {from 3}
5 D ∴ ~$\underline{T}$ {from 4}
6 D ∴ $\underline{C}$ {from 4}
7 asm: ~R$\underline{C}$ {break up 1}
8 ∴ O~$\underline{C}$ {from 7}
9 D ∴ ~$\underline{C}$ {from 8}
10 ∴ R$\underline{C}$ {from 7; 6 contradicts 9}
11 ∴ O$\underline{T}$ {from 1 and 10}
12 D ∴ $\underline{T}$ {from 11}
13 ∴ O($\underline{T}$ ∨ ~$\underline{C}$) {from 2; 5 contradicts 12}

13. Valid

1 O$\underline{S}$
* 2 ~◇(S • D)
[∴ R~$\underline{D}$
* 3 asm: ~R~$\underline{D}$
4 ∴ O$\underline{D}$ {from 3}
5 asm: ~O($\underline{S}$ • $\underline{D}$) {nice to have "O($\underline{S}$ • $\underline{D}$)" to use Kant's Law on to contradict 2}
6 ∴ R~($\underline{S}$ • $\underline{D}$) {from 5}
7 D ∴ ~($\underline{S}$ • $\underline{D}$) {from 6}
8 D ∴ $\underline{S}$ {from 1}
9 D ∴ $\underline{D}$ {from 4}
10 D ∴ ~$\underline{D}$ {from 7 and 8}
11 ∴ O($\underline{S}$ • $\underline{D}$) {from 5; 9 contradicts 10}
12 ∴ ◇(S • D) {from 11 using Kant's Law}
13 ∴ R~$\underline{D}$ {from 3; 2 contradicts 12}

14. Valid

1 O$\underline{H}$
2 ☐(H ⊃ (P ∨ A))
* 3 ~◇P
* 4 (◇A ⊃ G)
[∴ G
5 asm: ~G
* 6 ∴ ◇H {from 1 using Kant's Law}
7 ∴ ☐~P {from 3}
8 W ∴ H {from 6}
* 9 ∴ ~◇A {from 4 and 5}
10 ∴ ☐~A {from 9}
*11 W ∴ (H ⊃ (P ∨ A)) {from 2}
*12 W ∴ (P ∨ A) {from 8 and 11}
13 W ∴ ~P {from 7}
14 W ∴ A {from 12 and 13}
15 W ∴ ~A {from 10}
16 ∴ G {from 5; 14 contradicts 15}

16. Valid

* 1 (RA$\underline{u}$ ⊃ OA$\underline{u}$)
* 2 (OA$\underline{u}$ ⊃ O(x)A$\underline{x}$)
[∴ (~◇(x)Ax ⊃ O~A$\underline{u}$)
* 3 asm: ~(~◇(x)Ax ⊃ O~A$\underline{u}$)
* 4 ∴ ~◇(x)Ax {from 3}
* 5 ∴ ~O~A$\underline{u}$ {from 3}
6 ∴ RA$\underline{u}$ {from 5}
7 ∴ OA$\underline{u}$ {from 1 and 6}
8 ∴ O(x)A$\underline{x}$ {from 2 and 7}
9 ∴ ◇(x)Ax {from 8 by Kant's Law}
10 ∴ (~◇(x)A$\underline{x}$ ⊃ O~A$\underline{u}$) {from 3; 4 contradicts 9}

17. Invalid

1 O(∃x)(Bjx • H$\underline{s}$x)
[∴ O(∃x)B$\underline{j}$x
* 2 asm: ~O(∃x)B$\underline{j}$x
* 3 ∴ R~(∃x)B$\underline{j}$x {from 2}
* 4 D ∴ ~(∃x)B$\underline{j}$x {from 3}
5 D ∴ (x)~B$\underline{j}$x {from 4}
* 6 D ∴ (∃x)(Bjx • H$\underline{s}$x) {from 1}
* 7 D ∴ (Bja • H$\underline{s}$a) {from 6}
8 D ∴ Bja {from 7}
9 D ∴ H$\underline{s}$a {from 7}
10 D ∴ ~B$\underline{j}$a {from 5}
11 D ∴ ~B$\underline{j}$j {from 5}
12 D ∴ ~B$\underline{j}$s {from 5}
13 ∴ Bja {from 8 by indicative transfer}

18. Valid

* 1 (~R$\underline{A}$ ⊃ ~R$\underline{P}$)
[∴ ($\underline{P}$ ⊃ R$\underline{A}$)
* 2 asm: ~($\underline{P}$ ⊃ R$\underline{A}$)
3 ∴ $\underline{P}$ {from 2}
4 ∴ ~R$\underline{A}$ {from 2}
* 5 ∴ ~R$\underline{P}$ {from 1 and 4}
6 ∴ O~$\underline{P}$ {from 5}
7 ∴ ~$\underline{P}$ {from 6}
8 ∴ ($\underline{P}$ ⊃ R$\underline{A}$) {from 2; 3 contradicts 7}

19. Invalid

	a, b
D	A$\underline{b}$, ~A$\underline{a}$
DD	A$\underline{a}$, ~A$\underline{b}$

1 O(∃x)A$\underline{x}$
[∴ (∃x)OA$\underline{x}$
* 2 asm: ~(∃x)OA$\underline{x}$
3 ∴ (x)~OA$\underline{x}$ {from 2}
* 4 ∴ ~OA$\underline{a}$ {from 3}
* 5 ∴ R~A$\underline{a}$ {from 3}
6 D ∴ ~A$\underline{a}$ {from 5}
* 7 D ∴ (∃x)A$\underline{x}$ {from 1}
8 D ∴ A$\underline{b}$ {from 7}
* 9 ∴ ~OA$\underline{b}$ {from 3}
*10 ∴ R~A$\underline{b}$ {from 4}
11 DD ∴ ~A$\underline{b}$ {from 10}

Endless loop: add "A$\underline{a}$" to world DD to make the premise true. (Refutations aren't required in this exercise.)

21. Valid

* 1 (R$\underline{L}$ ⊃ O~$\underline{P}$)
[∴ ~R($\underline{P}$ • $\underline{L}$)
* 2 asm: R($\underline{P}$ • $\underline{L}$)
* 3 D ∴ ($\underline{P}$ • $\underline{L}$) {from 2}
4 D ∴ $\underline{P}$ {from 3}
5 D ∴ $\underline{L}$ {from 3}
6 asm: ~R$\underline{L}$ {break up 1}
7 ∴ O~$\underline{L}$ {from 6}
8 D ∴ ~$\underline{L}$ {from 7}
9 ∴ R$\underline{L}$ {from 6; 5 contradicts 8}
10 ∴ O~$\underline{P}$ {from 1 and 9}
11 D ∴ ~$\underline{P}$ {from 10}
12 ∴ ~R($\underline{P}$ • $\underline{L}$) {from 2; 4 contradicts 11}

22. Valid

* 1 (OB$\underline{u}$ ⊃ O(x)B$\underline{x}$)
* 2 ~◇(x)Bx
[∴ R~B$\underline{u}$
* 3 asm: ~R~B$\underline{u}$
4 ∴ ☐~(x)Bx {from 2}
5 ∴ OB$\underline{u}$ {from 3}

6 ∴ O(x)B$\underline{x}$ {from 1 and 5}
* 7 ∴ ~(x)Bx {from 4}
* 8 ∴ (∃x)~Bx {from 7}
9 ∴ ~Ba {from 8}
10 ∴ ◇(x)Bx {from 6 by Kant's Law}
11 ∴ R~B$\underline{u}$ {from 3; 2 contradicts 10}

23. Valid

1 O~($\underline{B}$ • $\underline{A}$)
2 O$\underline{B}$
[∴ O($\underline{B}$ • ~$\underline{A}$)
* 3 asm: ~O($\underline{B}$ • ~$\underline{A}$)
* 4 ∴ R~($\underline{B}$ • ~$\underline{A}$) {from 3}
* 5 D ∴ ~($\underline{B}$ • ~$\underline{A}$) {from 4}
* 6 D ∴ ~($\underline{B}$ • $\underline{A}$) {from 1}
7 D ∴ $\underline{B}$ {from 2}
8 D ∴ $\underline{A}$ {from 5 and 7}
9 D ∴ ~$\underline{A}$ {from 6 and 7}
10 ∴ O($\underline{B}$ • ~$\underline{A}$) {from 3; 8 contradicts 9}

24. Valid

* 1 ~◇(x)Bx
[∴ R(∃x)~B$\underline{x}$
* 2 asm: ~R(∃x)~B$\underline{x}$
3 ∴ □~(x)Bx {from 1}
4 ∴ O~(∃x)~B$\underline{x}$ {from 2}
* 5 ∴ ◇~(∃x)~Bx {from 4 using Kant's Law}
* 6 W ∴ ~(∃x)~Bx {from 5}
7 W ∴ (x)Bx {from 6}
8 W ∴ ~(x)Bx {from 3}
9 ∴ R(∃x)~B$\underline{x}$ {from 2; 7 contradicts 8}

26. Valid

1 O($\underline{C}$ ∨ $\underline{M}$)
* 2 (E ⊃ O~$\underline{M}$)
[∴ (E ⊃ O$\underline{C}$)
* 3 asm: ~(E ⊃ O$\underline{C}$)
4 ∴ E {from 3}
* 5 ∴ ~O$\underline{C}$ {from 3}
* 6 ∴ R~$\underline{C}$ {from 5}
7 D ∴ ~$\underline{C}$ {from 6}
8 ∴ O~$\underline{M}$ {from 2 and 4}
* 9 D ∴ ($\underline{C}$ ∨ $\underline{M}$) {from 1}
10 D ∴ $\underline{M}$ {from 7 and 9}
11 D ∴ ~$\underline{M}$ {from 8}
12 ∴ (E ⊃ O$\underline{C}$) {from 3; 10 contradicts 11}

27. Invalid

1 O$\underline{H}$
2 O(H ⊃ $\underline{S}$)
3 ~H
* 4 (~H ⊃ O~$\underline{S}$)
[∴ (O$\underline{S}$ • O~$\underline{S}$)
* 5 asm: ~(O$\underline{S}$ • O~$\underline{S}$)
6 ∴ O~$\underline{S}$ {from 3 and 4}
* 7 ∴ ~O$\underline{S}$ {from 5 and 6}
* 8 ∴ R~$\underline{S}$ {from 7}
9 D ∴ ~$\underline{S}$ {from 8}
10 D ∴ $\underline{H}$ {from 1}
* 11 D ∴ (H ⊃ $\underline{S}$) {from 2}
12 D ∴ ~H {from 9 and 11}

28. Invalid

1 (T ⊃ M)
2 O~$\underline{M}$
[∴ O~$\underline{T}$
* 3 asm: ~O~$\underline{T}$
* 4 ∴ R$\underline{T}$ {from 3}
5 D ∴ $\underline{T}$ {from 4}
6 D ∴ ~$\underline{M}$ {from 2}
7 asm: ~T {break up 1}

29. Valid

* 1 (O$\underline{U}$ ⊃ O$\underline{J}$)
2 ~◇(U • J)
[∴ ~O$\underline{U}$
3 asm: O$\underline{U}$
4 ∴ O$\underline{J}$ {from 1 and 3}
5 asm: ~O($\underline{U}$ • $\underline{J}$) {nice to have "O($\underline{U}$ • $\underline{J}$)" so we can use Kant's Law on it to contradict 2}
6 ∴ R~($\underline{U}$ • $\underline{J}$) {from 5}
7 D ∴ ~($\underline{U}$ • $\underline{J}$) {from 6}
8 D ∴ $\underline{U}$ {from 3}
9 D ∴ $\underline{J}$ {from 4}
10 D ∴ ~$\underline{J}$ {from 7 and 8}
11 ∴ O($\underline{U}$ • $\underline{J}$) {from 5; 9 contradicts 10}
12 ∴ ◇(U • J) {from 11 using Kant's Law}
13 ∴ ~O$\underline{U}$ {from 3; 2 contradicts 12}

10.1a

2. (~u:G • ~u:~G)
4. u:~◇G
6. $\underline{u}$:G
7. (~◇G ⊃ ~$\underline{u}$:G)
8. (u:A ⊃ ~u:~A)
9. (u:A ⊃ ~$\underline{u}$:~A)

10.2a

2. Invalid

* 1 $\sim\Diamond(A \cdot B)$
[$\therefore (u{:}A \supset \sim u{:}B)$
* 2 asm: $\sim(u{:}A \supset \sim u{:}B)$
3 $\therefore \Box\sim(A \cdot B)$ {from 1}
4 $\therefore u{:}A$ {from 2}
5 $\therefore u{:}B$ {from 2}

4. Valid

* 1 $\sim\Diamond(A \cdot B)$
[$\therefore (\sim\underline{u}{:}A \vee \sim\underline{u}{:}B)$
* 2 asm: $\sim(\sim\underline{u}{:}A \vee \sim\underline{u}{:}B)$
3 $\therefore \Box\sim(A \cdot B)$ {from 1}
4 $\therefore \underline{u}{:}A$ {from 2}
5 $\therefore \underline{u}{:}B$ {from 2}
6 $u \therefore A$ {from 4}
* 7 $u \therefore \sim(A \cdot B)$ {from 3}
8 $u \therefore \sim B$ {from 6 and 7}
9 $u \therefore B$ {from 5}
10 $\therefore (\sim\underline{u}{:}A \vee \sim\underline{u}{:}B)$ {from 2; 8 contradicts 9}

6. Invalid

1 $\Box(A \supset B)$
2 $u{:}A$
[$\therefore u{:}B$
3 asm: $\sim u{:}B$

7. Invalid

1 $\Box(A \supset B)$
2 $\underline{u}{:}A$
[$\therefore u{:}B$
3 asm: $\sim u{:}B$
4 $u \therefore A$ {from 2}
* 5 $u \therefore (A \supset B)$ {from 1}
6 $u \therefore B$ {from 4 and 5}

8. Valid

1 $\Box(A \supset B)$
* 2 $\sim\underline{u}{:}\sim A$
[$\therefore \sim\underline{u}{:}\sim B$
3 asm: $\underline{u}{:}\sim B$
4 $u \therefore A$ {from 2}
* 5 $u \therefore (A \supset B)$ {from 1}
6 $u \therefore B$ {from 4 and 5}
7 $u \therefore \sim B$ {from 3}
8 $\therefore \sim\underline{u}{:}\sim B$ {from 3; 6 contradicts 7}

9. Invalid

1 $\Box(A \supset B)$
* 2 $\sim\underline{u}{:}B$
[$\therefore \underline{u}{:}\sim A$
* 3 asm: $\sim\underline{u}{:}\sim A$
4 $u \therefore \sim B$ {from 2}
5 $uu \therefore A$ {from 3}
* 6 $u \therefore (A \supset B)$ {from 1}
7 $u \therefore \sim A$ {from 4 and 6}
* 8 $uu \therefore (A \supset B)$ {from 1}
9 $uu \therefore B$ {from 5 and 8}

10.2b

2. Invalid

1 $u{:}A$
[$\therefore \sim u{:}\sim A$
2 asm: $u{:}\sim A$

4. Valid

[$\therefore (\sim\Diamond A \supset \sim\underline{u}{:}A)$
* 1 asm: $\sim(\sim\Diamond A \supset \sim\underline{u}{:}A)$
2 $\therefore \sim\Diamond A$ {from 1}
3 $\therefore \underline{u}{:}A$ {from 1}
4 $\therefore \Box\sim A$ {from 2}
5 $u \therefore A$ {from 3}
6 $u \therefore \sim A$ {from 4}
7 $\therefore (\sim\Diamond A \supset \sim\underline{u}{:}A)$ {from 1; 5 contradicts 6}

6. Valid

1 $\underline{u}{:}A$
[$\therefore \sim\underline{u}{:}\sim A$
2 asm: $\underline{u}{:}\sim A$
3 $u \therefore A$ {from 1}
4 $u \therefore \sim A$ {from 2}
5 $\therefore \sim\underline{u}{:}\sim A$ {from 2; 3 contradicts 4}

7. Valid

[$\therefore \sim(\underline{u}{:}A \cdot \sim\underline{u}{:}\Diamond A)$
* 1 asm: $(\underline{u}{:}A \cdot \sim\underline{u}{:}\Diamond A)$
2 $\therefore \underline{u}{:}A$ {from 1}
* 3 $\therefore \sim\underline{u}{:}\Diamond A$ {from 1}
* 4 $u \therefore \sim\Diamond A$ {from 3}
5 $u \therefore \Box\sim A$ {from 4}
6 $u \therefore A$ {from 2}
7 $u \therefore \sim A$ {from 5}
8 $\therefore \sim(\underline{u}{:}A \cdot \sim\underline{u}{:}\Diamond A)$ {from 1; 6 contradicts 7}

8. Valid

1 $\Box((A \cdot B) \supset C)$
[$\therefore \sim((\underline{u}{:}A \cdot \underline{u}{:}B) \cdot \sim\underline{u}{:}C)$
* 2 asm: $((\underline{u}{:}A \cdot \underline{u}{:}B) \cdot \sim\underline{u}{:}C)$
* 3 $\therefore (\underline{u}{:}A \cdot \underline{u}{:}B)$ {from 2}

* 4 ∴ ~$\underline{u}$:C {from 2}
5 ∴ $\underline{u}$:A {from 3}
6 ∴ $\underline{u}$:B {from 3}
7 u ∴ ~C {from 4}
* 8 u ∴ ((A • B) ⊃ C) {from 1}
* 9 u ∴ ~(A • B) {from 7 and 8}
10 u ∴ A {from 5}
11 u ∴ ~B {from 9 and 10}
12 u ∴ B {from 6}
13 ∴ ~(($\underline{u}$:A • $\underline{u}$:B) • ~$\underline{u}$:C) {from 2; 11 contradicts 12}

9. Invalid

1 □(A ⊃ (B • C))
* 2 ~$\underline{u}$:B
[∴ $\underline{u}$:~A
* 3 asm: ~$\underline{u}$:~A
4 u ∴ ~B {from 2}
5 uu ∴ A {from 3}
6 u ∴ (A ⊃ (B • C)) {from 1}
* 7 uu ∴ (A ⊃ (B • C)) {from 1}
* 8 uu ∴ (B • C) {from 5 and 7}
9 uu ∴ B {from 8}
10 uu ∴ C {from 8}
11 u asm: ~A {break up 6}

10.3a

2. u:OS$\underline{j}$
4. u:(x)~E$\underline{u}$x
6. (u:K$\underline{u}$ • ~Ku)
7. (u:OA$\underline{u}$ ⊃ A$\underline{u}$)
8. ~($\underline{u}$:OA$\underline{u}$ • ~$\underline{u}$:A$\underline{u}$)
9. ~$\underline{u}$:(x)~E$\underline{u}$x
11. u:(N ⊃ ~K$\underline{u}$)
12. (A$\underline{u}$ ⊃ u:(x)A$\underline{u}$)
13. (Axu ⊃ A$\underline{u}$x)
14. (Aux ⊃ Axu)

10.4a

2. Valid

[∴ $\underline{u}$:(B$\underline{a}$ ⊃ RB$\underline{a}$)
* 1 asm: ~$\underline{u}$:(B$\underline{a}$ ⊃ RB$\underline{a}$)
* 2 u ∴ ~(B$\underline{a}$ ⊃ RB$\underline{a}$) {from 1}
3 u ∴ B$\underline{a}$ {from 2}
* 4 u ∴ ~RB$\underline{a}$ {from 2}
5 u ∴ O~B$\underline{a}$ {from 4}
6 u ∴ ~B$\underline{a}$ {from 5}
7 ∴ $\underline{u}$:(B$\underline{a}$ ⊃ RB$\underline{a}$) {from 1; 3 contradicts 6}

4. Valid

[∴ ~(($\underline{u}$:(A ⊃ $\underline{B}$) • $\underline{u}$:A) • ~$\underline{u}$:$\underline{B}$)
* 1 asm: (($\underline{u}$:(A ⊃ $\underline{B}$) • $\underline{u}$:A) • ~$\underline{u}$:$\underline{B}$)
* 2 ∴ ($\underline{u}$:(A ⊃ $\underline{B}$) • $\underline{u}$:A) {from 1
* 3 ∴ ~$\underline{u}$:$\underline{B}$ {from 1}
4 ∴ $\underline{u}$:(A ⊃ $\underline{B}$) {from 2}
5 ∴ $\underline{u}$:A {from 2}
6 u ∴ ~$\underline{B}$ {from 3}
* 7 u ∴ (A ⊃ $\underline{B}$) {from 4}
8 u ∴ ~A {from 6 and 7}
9 u ∴ A {from 5}
10 ∴ ~(($\underline{u}$:(A ⊃ $\underline{B}$) • $\underline{u}$:A) • ~$\underline{u}$:$\underline{B}$) {from 1; 8 contradicts 9}

6. Invalid

1 ~u:A$\underline{u}$
[∴ ~$\underline{u}$:OA$\underline{u}$
2 asm: $\underline{u}$:OA$\underline{u}$
3 u ∴ OA$\underline{u}$ {from 2}

7. Valid

[∴ $\underline{u}$:(OA$\underline{u}$ ⊃ A$\underline{u}$)
* 1 asm: ~$\underline{u}$:(OA$\underline{u}$ ⊃ A$\underline{u}$)
* 2 u ∴ ~(OA$\underline{u}$ ⊃ A$\underline{u}$) {from 1}
3 u ∴ OA$\underline{u}$ {from 2}
4 u ∴ ~A$\underline{u}$ {from 2}
5 u ∴ A$\underline{u}$ {from 3}
6 ∴ $\underline{u}$:(OA$\underline{u}$ ⊃ A$\underline{u}$) {from 1; 4 contradicts 5}

8. Valid

[∴ ($\underline{u}$:A$\underline{u}$ ∨ ~$\underline{u}$:OA$\underline{u}$)
* 1 asm: ~($\underline{u}$:A$\underline{u}$ ∨ ~$\underline{u}$:OA$\underline{u}$)
* 2 ∴ ~$\underline{u}$:A$\underline{u}$ {from 1}
3 ∴ $\underline{u}$:OA$\underline{u}$ {from 1}
4 u ∴ ~A$\underline{u}$ {from 2}
5 u ∴ OA$\underline{u}$ {from 3}
6 u ∴ A$\underline{u}$ {from 5}
7 ∴ ($\underline{u}$:A$\underline{u}$ ∨ ~$\underline{u}$:OA$\underline{u}$) {from 1; 4 contradicts 6}

9. Invalid

1 u:A$\underline{u}$
[∴ ~$\underline{u}$:O~A$\underline{u}$
2 asm: $\underline{u}$:O~A$\underline{u}$
3 u ∴ O~A$\underline{u}$ {from 2}

10.4b

2. Valid

[∴ ~($\underline{u}$:OA$\underline{u}$ • ~$\underline{u}$:A$\underline{u}$)

* 1 asm: ($\underline{u}$:OA$\underline{u}$ • ~$\underline{u}$:A$\underline{u}$)
2 ∴ $\underline{u}$:OA$\underline{u}$ {from 1}
* 3 ∴ ~$\underline{u}$:A$\underline{u}$ {from 1}
4 u ∴ ~A$\underline{u}$ {from 3}
5 u ∴ OA$\underline{u}$ {from 2}
6 u ∴ A$\underline{u}$ {from 5}
7 ∴ ~($\underline{u}$:OA$\underline{u}$ • ~$\underline{u}$:A$\underline{u}$) {from 1; 4 contradicts 6}

4. Valid

[∴ ~($\underline{u}$:(x)OA$\underline{x}$ • ~$\underline{u}$:A$\underline{u}$)
* 1 asm: ($\underline{u}$:(x)OA$\underline{x}$ • ~$\underline{u}$:A$\underline{u}$)
2 ∴ $\underline{u}$:(x)OA$\underline{x}$ {from 1}
* 3 ∴ ~$\underline{u}$:A$\underline{u}$ {from 1}
4 u ∴ ~A$\underline{u}$ {from 3}
5 u ∴ (x)OA$\underline{x}$ {from 2}
6 u ∴ OA$\underline{u}$ {from 5}
7 u ∴ A$\underline{u}$ {from 6}
8 ∴ ~($\underline{u}$:(x)OA$\underline{x}$ • ~$\underline{u}$:A$\underline{u}$) {from 1; 4 contradicts 7}

6. Invalid

1 ☐($\underline{E}$ ⊃ (N ⊃ $\underline{M}$))
[∴ ((u:$\underline{E}$ • u:N) ⊃ $\underline{M}$)
* 2 asm: ~((u:$\underline{E}$ • u:N) ⊃ $\underline{M}$)
* 3 ∴ (u:$\underline{E}$ • u:N) {from 2}
4 ∴ ~$\underline{M}$ {from 2}
5 ∴ u:$\underline{E}$ {from 3}
6 ∴ u:N {from 3}

7. Valid

[∴ ~($\underline{u}$:(x)O~K$\underline{x}$ • ~$\underline{u}$:(N ⊃ ~K$\underline{u}$))
* 1 asm: ($\underline{u}$:(x)O~K$\underline{x}$ • ~$\underline{u}$:(N ⊃ ~K$\underline{u}$))
2 ∴ $\underline{u}$:(x)O~K$\underline{x}$ {from 1}
* 3 ∴ ~$\underline{u}$:(N ⊃ ~K$\underline{u}$) {from 1}
* 4 u ∴ ~(N ⊃ ~K$\underline{u}$) {from 3}
5 u ∴ N {from 4}
6 u ∴ K$\underline{u}$ {from 4}
7 u ∴ (x)O~K$\underline{x}$ {from 2}
8 u ∴ O~K$\underline{u}$ {from 7}
9 u ∴ ~K$\underline{u}$ {from 8}
10 ∴ ~($\underline{u}$:(x)O~K$\underline{x}$ • ~$\underline{u}$:(N ⊃ ~K$\underline{u}$)) {from 1; 6 contradicts 9}

8. Invalid. The "⊃" poorly translates the contrary-to-fact conditional "If killing were needed to save your family then you wouldn't kill"; but the argument would be invalid even if this were formulated more adequately.

[∴ ~($\underline{u}$:(x)O~K$\underline{x}$ • ~(N ⊃ ~Ku))
* 1 asm: ($\underline{u}$:(x)O~K$\underline{x}$ • ~(N ⊃ ~Ku))
2 ∴ $\underline{u}$:(x)O~K$\underline{x}$ {from 1}
* 3 ∴ ~(N ⊃ ~Ku) {from 1}
4 ∴ N {from 3}
5 ∴ Ku {from 3}
6 u ∴ (x)O~K$\underline{x}$ {from 2}
7 u ∴ O~K$\underline{u}$ {from 6}

9. Valid

[∴ ~($\underline{u}$:O~A$\underline{j}$ • $\underline{u}$:A$\underline{j}$)
* 1 asm: ($\underline{u}$:O~A$\underline{j}$ • $\underline{u}$:A$\underline{j}$)
2 ∴ $\underline{u}$:O~A$\underline{j}$ {from 1}
3 ∴ $\underline{u}$:A$\underline{j}$ {from 1}
4 u ∴ O~A$\underline{j}$ {from 2}
5 u ∴ A$\underline{j}$ {from 3}
6 u ∴ ~A$\underline{j}$ {from 4}
7 ∴ ~($\underline{u}$:O~A$\underline{j}$ • $\underline{u}$:A$\underline{j}$) {from 1; 5 contradicts 6}

11. Valid

[∴ ~($\underline{u}$:A$\underline{u}$ • ~$\underline{u}$:RA$\underline{u}$)
* 1 asm: ($\underline{u}$:A$\underline{u}$ • ~$\underline{u}$:RA$\underline{u}$)
2 ∴ $\underline{u}$:A$\underline{u}$ {from 1}
* 3 ∴ ~$\underline{u}$:RA$\underline{u}$ {from 1}
* 4 u ∴ ~RA$\underline{u}$ {from 3}
5 u ∴ O~A$\underline{u}$ {from 4}
6 u ∴ A$\underline{u}$ {from 2}
7 u ∴ ~A$\underline{u}$ {from 5}
8 ∴ ~($\underline{u}$:A$\underline{u}$ • ~$\underline{u}$:RA$\underline{u}$) {from 1; 6 contradicts 7}

12. Invalid

[∴ (u:A$\underline{u}$ ⊃ $\underline{u}$:RA$\underline{u}$)
* 1 asm: ~(u:A$\underline{u}$ ⊃ $\underline{u}$:RA$\underline{u}$)
2 ∴ u:A$\underline{u}$ {from 1}
* 3 ∴ ~$\underline{u}$:RA$\underline{u}$ {from 1}
* 4 u ∴ ~RA$\underline{u}$ {from 3}
5 u ∴ O~A$\underline{u}$ {from 4}

13. Invalid

[∴ ~($\underline{u}$:A$\underline{u}$ • ~$\underline{u}$:OA$\underline{u}$)
* 1 asm: ($\underline{u}$:A$\underline{u}$ • ~$\underline{u}$:OA$\underline{u}$)
2 ∴ $\underline{u}$:A$\underline{u}$ {from 1}
* 3 ∴ ~$\underline{u}$:OA$\underline{u}$ {from 1}
* 4 u ∴ ~OA$\underline{u}$ {from 3}
* 5 u ∴ R~A$\underline{u}$ {from 4}
6 uD ∴ ~A$\underline{u}$ {from 5}
7 u ∴ A$\underline{u}$ {from 2}

14. Valid

1 $\underline{u}$:OA$\underline{u}$

[∴ $\underline{u}$:A$\underline{u}$
* 2 asm: ~$\underline{u}$:A$\underline{u}$
3 u ∴ ~A$\underline{u}$ {from 2}
4 u ∴ OA$\underline{u}$ {from 1}
5 u ∴ A$\underline{u}$ {from 4}
6 ∴ $\underline{u}$:A$\underline{u}$ {from 2; 3 contradicts 5}

10.5a

2. O$\underline{u}$:Sj
4. R$\underline{u}$:G
6. ~R$\underline{u}$:(x)~R$\underline{x}$:G
7. (R$\underline{u}$:G ⊃ ◇G)
8. O~($\underline{u}$:G • ~$\underline{u}$:◇G)
9. O~($\underline{u}$:OA$\underline{u}$ • ~$\underline{u}$:A$\underline{u}$)
11. (R(~$\underline{u}$:G • ~$\underline{u}$:~G) ⊃ ~O$\underline{u}$:G)
12. (u:A • A)
13. (u:A • ~A)
14. ~◇(u:A • ~A)
16. ◇((u:A • O$\underline{u}$:A) • ~A)
17. (x)O$\underline{x}$:(Dx ⊃ Ex)
18. ((□(A ⊃ B) • ~R$\underline{u}$:B) ⊃ ~R$\underline{u}$:A)
19. (RA$\underline{u}$ ⊃ u:(x)A$\underline{x}$)
21. O~($\underline{u}$:A$\underline{u}$x • $\underline{u}$:~A$\underline{x}$u)
22. □(Pu ⊃ O$\underline{u}$:Ou)
23. □(u:x=x ⊃ O$\underline{u}$:x=x)
24. ((~Du • O$\underline{u}$:u:Su) ⊃ O$\underline{u}$:R)

10.6a

2. Invalid

1 O~$\underline{u}$:A
[∴ O$\underline{u}$:~A
* 2 asm: ~O$\underline{u}$:~A
* 3 ∴ R~$\underline{u}$:~A {from 2}
* 4 D ∴ ~$\underline{u}$:~A {from 3}
5 Du ∴ A {from 4}
* 6 D ∴ ~$\underline{u}$:A {from 1}
7 Duu ∴ ~A {from 6}

4. Valid

* 1 R$\underline{u}$:~A
[∴ R~$\underline{u}$:A
* 2 asm: ~R~$\underline{u}$:A
3 D ∴ $\underline{u}$:~A {from 1}
4 ∴ O$\underline{u}$:A {from 2}
5 D ∴ $\underline{u}$:A {from 4}
6 Du ∴ ~A {from 3}
7 Du ∴ A {from 5}
8 ∴ R~$\underline{u}$:A {from 2; 6 contradicts 7}

6. Valid

[∴ O~($\underline{u}$:A • ~$\underline{u}$:◇A)
* 1 asm: ~O~($\underline{u}$:A • ~$\underline{u}$:◇A)
* 2 ∴ R($\underline{u}$:A • ~$\underline{u}$:◇A) {from 1}
* 3 D ∴ ($\underline{u}$:A • ~$\underline{u}$:◇A) {from 2}
4 D ∴ $\underline{u}$:A {from 3}
* 5 D ∴ ~$\underline{u}$:◇A {from 3}
* 6 Du ∴ ~◇A {from 5}
7 Du ∴ □~A {from 6}
8 Du ∴ A {from 4}
9 Du ∴ ~A {from 7}
10 ∴ O~($\underline{u}$:A • ~$\underline{u}$:◇A) {from 1; 8 contradicts 9}

7. Valid

[∴ (R$\underline{u}$:A ⊃ ◇A)
* 1 asm: ~(R$\underline{u}$:A ⊃ ◇A)
* 2 ∴ R$\underline{u}$:A {from 1}
* 3 ∴ ~◇A {from 1}
4 D ∴ $\underline{u}$:A {from 2}
5 ∴ □~A {from 3}
6 Du ∴ A {from 4}
7 Du ∴ ~A {from 5}
8 ∴ (R$\underline{u}$:A ⊃ ◇A) {from 1; 6 contradicts 7}

8. Invalid

1 □(A ⊃ B)
[∴ (R~$\underline{u}$:B ⊃ R$\underline{u}$:~A)
* 2 asm: ~(R~$\underline{u}$:B ⊃ R$\underline{u}$:~A)
* 3 ∴ R~$\underline{u}$:B {from 2}
* 4 ∴ ~R$\underline{u}$:~A {from 2}
* 5 D ∴ ~$\underline{u}$:B {from 3}
6 ∴ O~$\underline{u}$:~A {from 4}
7 Du ∴ ~B {from 5}
* 8 Du ∴ (A ⊃ B) {from 1}
9 Du ∴ ~A {from 7 and 8}
* 10 D ∴ ~$\underline{u}$:~A {from 6}
11 Duu ∴ A {from 10}
* 12 Duu ∴ (A ⊃ B) {from 1}
13 Duu ∴ B {from 11 and 12}

9. Valid

* 1 R$\underline{u}$:OA$\underline{u}$
[∴ R$\underline{u}$:◇Au
* 2 asm: ~R$\underline{u}$:◇Au
3 D ∴ $\underline{u}$:OA$\underline{u}$ {from 1}
4 ∴ O~$\underline{u}$:◇Au {from 2}
* 5 D ∴ ~$\underline{u}$:◇Au {from 4}
* 6 Du ∴ ~◇Au {from 5}
7 Du ∴ OA$\underline{u}$ {from 3}

8 Du ∴ ◇Au {from 7 by Kant's Law}
9 ∴ R$\underline{u}$:◇Au {from 2; 6 contradicts 8}

10.6b

2. Invalid

* 1 ~O$\underline{u}$:G
[∴ R$\underline{u}$:~G
* 2 asm: ~R$\underline{u}$:~G
* 3 ∴ R~$\underline{u}$:G {from 1}
4 ∴ O~$\underline{u}$:~G {from 2}
* 5 D ∴ ~$\underline{u}$:G {from 3}
6 Du ∴ ~G {from 5}
* 7 D ∴ ~$\underline{u}$:~G {from 4}
8 Duu ∴ G {from 7}

4. Invalid

[∴ (u:OA$\underline{u}$ ⊃ OA$\underline{u}$)
* 1 asm: ~(u:OA$\underline{u}$ ⊃ OA$\underline{u}$)
2 ∴ u:OA$\underline{u}$ {from 1}
* 3 ∴ ~OA$\underline{u}$ {from 1}
* 4 ∴ R~A$\underline{u}$ {from 3}
5 D ∴ ~A$\underline{u}$ {from 4}

6. Invalid

* 1 R$\underline{u}$:A
* 2 R$\underline{u}$:B
[∴ R$\underline{u}$:(A • B)
* 3 asm: ~R$\underline{u}$:(A • B)
4 D ∴ $\underline{u}$:A {from 1}
5 DD ∴ $\underline{u}$:B {from 2}
6 ∴ O~$\underline{u}$:(A • B) {from 3}
* 7 D ∴ ~$\underline{u}$:(A • B) {from 6}
* 8 Du ∴ ~(A • B) {from 7}
9 Du ∴ A {from 4}
10 Du ∴ ~B {from 8 and 9}
11 DD ∴ ~$\underline{u}$:(A • B) {from 6}
12 DDu ∴ B {from 5}

7. Valid

1 O$\underline{i}$:(H ⊃ ~P)
* 2 ~O$\underline{i}$:~H
[∴ ~O$\underline{i}$:P
3 asm: O$\underline{i}$:P
* 4 ∴ R~$\underline{i}$:~H {from 2}
* 5 D ∴ ~$\underline{i}$:~H {from 4}
6 Di ∴ H {from 5}
7 D ∴ $\underline{i}$:(H ⊃ ~P) {from 1}
8 D ∴ $\underline{i}$:P {from 3}
* 9 Di ∴ (H ⊃ ~P) {from 7}
10 Di ∴ ~P {from 6 and 9}
11 Di ∴ P {from 8}
12 ∴ ~O$\underline{i}$:P {from 3; 10 contradicts 11}

8. Valid

1 O$\underline{i}$:(H ⊃ ~P)
* 2 (~D ⊃ O$\underline{i}$:P)
3 ~D
[∴ O$\underline{i}$:~H
* 4 asm: ~O$\underline{i}$:~H
* 5 ∴ R~$\underline{i}$:~H {from 4}
* 6 D ∴ ~$\underline{i}$:~H {from 5}
7 Di ∴ H {from 6}
8 ∴ O$\underline{i}$:P {from 2 and 3}
9 D ∴ $\underline{i}$:(H ⊃ ~P) {from 1}
10 D ∴ $\underline{i}$:P {from 8}
* 11 Di ∴ (H ⊃ ~P) {from 9}
12 Di ∴ ~P {from 7 and 11}
13 Di ∴ P {from 10}
14 ∴ O$\underline{i}$:~H {from 4; 12 contradicts 13}

9. Valid

1 O$\underline{u}$:N
2 □($\underline{E}$ ⊃ (N ⊃ $\underline{M}$))
[∴ O~($\underline{u}$:$\underline{E}$ • ~$\underline{u}$:$\underline{M}$)
* 3 asm: ~O~($\underline{u}$:$\underline{E}$ • ~$\underline{u}$:$\underline{M}$)
* 4 ∴ R($\underline{u}$:$\underline{E}$ • ~$\underline{u}$:$\underline{M}$) {from 3}
* 5 D ∴ ($\underline{u}$:$\underline{E}$ • ~$\underline{u}$:$\underline{M}$) {from 4}
6 D ∴ $\underline{u}$:$\underline{E}$ {from 5}
* 7 D ∴ ~$\underline{u}$:$\underline{M}$ {from 5}
8 Du ∴ ~$\underline{M}$ {from 7}
9 D ∴ $\underline{u}$:N {from 1}
* 10 Du ∴ ($\underline{E}$ ⊃ (N ⊃ $\underline{M}$)) {from 2}
11 Du ∴ $\underline{E}$ {from 6}
* 12 Du ∴ (N ⊃ $\underline{M}$) {from 10 and 11}
13 Du ∴ ~N {from 8 and 12}
14 Du ∴ N {from 9}
15 ∴ O~($\underline{u}$:$\underline{E}$ • ~$\underline{u}$:$\underline{M}$) {from 3; 13 contradicts 14}

11. Valid

1 O$\underline{u}$:(RH$\underline{u}$j ⊃ RH$\underline{j}$u)
[∴ ~($\underline{u}$:H$\underline{u}$j • $\underline{u}$:O~H$\underline{j}$u)
* 2 asm: ($\underline{u}$:H$\underline{u}$j • $\underline{u}$:O~H$\underline{j}$u)
3 ∴ $\underline{u}$:H$\underline{u}$j {from 2}
4 ∴ $\underline{u}$:O~H$\underline{j}$u {from 2}
5 ∴ $\underline{u}$:(RH$\underline{u}$j ⊃ RH$\underline{j}$u) {from 1}
6 u ∴ H$\underline{u}$j {from 3}
7 u ∴ O~H$\underline{j}$u {from 4}
* 8 u ∴ (RH$\underline{u}$j ⊃ RH$\underline{j}$u) {from 5}

9 u ∴ ~H$\underline{j}$u {from 7}
10 u asm: ~RH$\underline{u}$j {break up 8}
11 u ∴ O~H$\underline{u}$j {from 10}
12 u ∴ ~H$\underline{u}$j {from 11}
13 u ∴ RH$\underline{u}$j {from 10; 6 contradicts 12}
*14 u ∴ RH$\underline{j}$u {from 8 and 13}
15 uD ∴ H$\underline{j}$u {from 14}
16 uD ∴ ~H$\underline{j}$u {from 7}
17 ∴ ~($\underline{u}$:H$\underline{u}$j • $\underline{u}$:O~H$\underline{j}$u) {from 2; 15 contradicts 16}

12. Valid

[∴ (R$\underline{u}$:$\underline{A}$ ⊃ R$\underline{u}$:R$\underline{A}$)
* 1 asm: ~(R$\underline{u}$:$\underline{A}$ ⊃ R$\underline{u}$:R$\underline{A}$)
* 2 ∴ R$\underline{u}$:$\underline{A}$ {from 1}
* 3 ∴ ~R$\underline{u}$:R$\underline{A}$ {from 1}
4 D ∴ $\underline{u}$:$\underline{A}$ {from 2}
5 ∴ O~$\underline{u}$:R$\underline{A}$ {from 3}
* 6 D ∴ ~$\underline{u}$:R$\underline{A}$ {from 5}
* 7 Du ∴ ~R$\underline{A}$ {from 6}
8 Du ∴ O~$\underline{A}$ {from 7}
9 Du ∴ $\underline{A}$ {from 4}
10 Du ∴ ~$\underline{A}$ {from 8}
11 ∴ (R$\underline{u}$:$\underline{A}$ ⊃ R$\underline{u}$:R$\underline{A}$) {from 1; 9 contradicts 10}

13. Invalid

1 O$\underline{u}$:A
[∴ A
2 asm: ~A
3 ∴ $\underline{u}$:A {from 1}
4 u ∴ A {from 3}

14. Valid

* 1 R$\underline{u}$:(G • T)
2 □(T ⊃ E)
[∴ R$\underline{u}$:(G • E)
* 3 asm: ~R$\underline{u}$:(G • E)
4 D ∴ $\underline{u}$:(G • T) {from 1}
5 ∴ O~$\underline{u}$:(G • E) {from 3}
* 6 D ∴ ~$\underline{u}$:(G • E) {from 5}
* 7 Du ∴ ~(G • E) {from 6}
* 8 Du ∴ (T ⊃ E) {from 2}
* 9 Du ∴ (G • T) {from 4}
10 Du ∴ G {from 9}
11 Du ∴ T {from 9}
12 Du ∴ ~E {from 7 and 10}
13 Du ∴ E {from 8 and 11}
14 ∴ R$\underline{u}$:(G • E) {from 3; 12 contradicts 13}

16. Invalid

* 1 R$\underline{u}$:G
[∴ ~R$\underline{u}$:~G
* 2 asm: R$\underline{u}$:~G
3 D ∴ $\underline{u}$:G {from 1}
4 DD ∴ $\underline{u}$:~G {from 2}
5 Du ∴ G {from 3}
6 DDu ∴ ~G {from 4}

17. Valid

1 O$\underline{u}$:G
[∴ ~R(~$\underline{u}$:G • ~$\underline{u}$:~G)
* 2 asm: R(~$\underline{u}$:G • ~$\underline{u}$:~G)
* 3 D ∴ (~$\underline{u}$:G • ~$\underline{u}$:~G) {from 2}
* 4 D ∴ ~$\underline{u}$:G {from 3}
* 5 D ∴ ~$\underline{u}$:~G {from 3}
6 Du ∴ ~G {from 4}
7 Duu ∴ G {from 5}
8 D ∴ $\underline{u}$:G {from 1}
9 ∴ ~R(~$\underline{u}$:G • ~$\underline{u}$:~G) {from 2; 4 contradicts 8}

18. Valid

[∴ (R$\underline{u}$:G ⊃ ◇G)
* 1 asm: ~(R$\underline{u}$:G ⊃ ◇G)
* 2 ∴ R$\underline{u}$:G {from 1}
* 3 ∴ ~◇G {from 1}
4 D ∴ $\underline{u}$:G {from 2}
5 ∴ □~G {from 3}
6 Du ∴ G {from 4}
7 Du ∴ ~G {from 5}
8 ∴ (R$\underline{u}$:G ⊃ ◇G) {from 1; 6 contradicts 7}

19. Valid

[∴ (~R$\underline{u}$:A ⊃ ~$\underline{u}$:A)
* 1 asm: ~(~R$\underline{u}$:A ⊃ ~$\underline{u}$:A)
* 2 ∴ ~R$\underline{u}$:A {from 1}
3 ∴ $\underline{u}$:A {from 1}
4 ∴ O~$\underline{u}$:A {from 2}
5 ∴ ~$\underline{u}$:A {from 4}
6 ∴ (~R$\underline{u}$:A ⊃ ~$\underline{u}$:A) {from 1; 3 contradicts 5}

21. Invalid

[∴ O~($\underline{u}$:~$\underline{A}$ • $\underline{u}$:R$\underline{A}$)
* 1 asm: ~O~($\underline{u}$:~$\underline{A}$ • $\underline{u}$:R$\underline{A}$)
* 2 ∴ R($\underline{u}$:~$\underline{A}$ • $\underline{u}$:R$\underline{A}$) {from 1}
* 3 D ∴ ($\underline{u}$:~$\underline{A}$ • $\underline{u}$:R$\underline{A}$) {from 2}
4 D ∴ $\underline{u}$:~$\underline{A}$ {from 3}
5 D ∴ $\underline{u}$:R$\underline{A}$ {from 3}

6 Du ∴ ~$\underline{A}$ {from 4}
* 7 Du ∴ R$\underline{A}$ {from 5}
8 DuD ∴ $\underline{A}$ {from 7}

22. Invalid

* 1 R~$\underline{u}$:E
[∴ R$\underline{u}$:~E
* 2 asm: ~R$\underline{u}$:~E
* 3 D ∴ ~$\underline{u}$:E {from 1}
4 ∴ O~$\underline{u}$:~E {from 2}
5 Du ∴ ~E {from 3}
* 6 D ∴ ~$\underline{u}$:~E {from 4}
7 Duu ∴ E {from 6}

23. Valid

* 1 R$\underline{u}$:O$\underline{A}$
[∴ R$\underline{u}$:$\underline{A}$
* 2 asm: ~R$\underline{u}$:$\underline{A}$
3 D ∴ $\underline{u}$:O$\underline{A}$ {from 1}
4 ∴ O~$\underline{u}$:$\underline{A}$ {from 2}
* 5 D ∴ ~$\underline{u}$:$\underline{A}$ {from 4}
6 Du ∴ ~$\underline{A}$ {from 5}
7 Du ∴ O$\underline{A}$ {from 3}
8 Du ∴ $\underline{A}$ {from 7}
9 ∴ R$\underline{u}$:$\underline{A}$ {from 2; 6 contradicts 8}

24. Invalid

[∴ (R$\underline{u}$:G ∨ R$\underline{u}$:~G)
* 1 asm: ~(R$\underline{u}$:G ∨ R$\underline{u}$:~G)
* 2 ∴ ~R$\underline{u}$:G {from 1}
* 3 ∴ ~R$\underline{u}$:~G {from 1}
4 ∴ O~$\underline{u}$:G {from 2}
5 ∴ O~$\underline{u}$:~G {from 3}
* 6 ∴ ~$\underline{u}$:G {from 4}
7 u ∴ ~G {from 6}
* 8 ∴ ~$\underline{u}$:~G {from 5}
9 uu ∴ G {from 8}

26. Valid

1 ☐(A ⊃ B)
* 2 R$\underline{u}$:A
[∴ R$\underline{u}$:B
* 3 asm: ~R$\underline{u}$:B
4 D ∴ $\underline{u}$:A {from 2}
5 ∴ O~$\underline{u}$:B {from 3}
* 6 D ∴ ~$\underline{u}$:B {from 5}
7 Du ∴ ~B {from 6}
* 8 Du ∴ (A ⊃ B) {from 1}
9 Du ∴ ~A {from 7and 8}
10 Du ∴ A {from 4}
11 ∴ R$\underline{u}$:B {from 3; 9 contradicts 10}

27. Invalid

* 1 ~O$\underline{u}$:~G
[∴ R$\underline{u}$:G
* 2 asm: ~R$\underline{u}$:G
* 3 ∴ R~$\underline{u}$:~G {from 1}
4 ∴ O~$\underline{u}$:G {from 2}
* 5 D ∴ ~$\underline{u}$:~G {from 3}
6 Du ∴ G {from 5}
* 7 D ∴ ~$\underline{u}$:G {from 4}
8 Duu ∴ ~G {from 7}

28. Valid

* 1 ~R$\underline{u}$:~G
* 2 ~R(~$\underline{u}$:G • ~$\underline{u}$:~G)
[∴ O$\underline{u}$:G
* 3 asm: ~O$\underline{u}$:G
4 ∴ O~$\underline{u}$:~G {from 1}
5 ∴ O~(~$\underline{u}$:G • ~$\underline{u}$:~G) {from 2}
* 6 ∴ R~$\underline{u}$:G {from 3}
* 7 D ∴ ~$\underline{u}$:G {from 6}
8 Du ∴ ~G {from 7}
* 9 D ∴ ~$\underline{u}$:~G {from 4}
10 Duu ∴ G {from 9}
* 11 D ∴ ~(~$\underline{u}$:G • ~$\underline{u}$:~G) {from 5}
12 D ∴ $\underline{u}$:~G {from 7 and 11}
13 ∴ O$\underline{u}$:G {from 3; 9 contradicts 12}

29. Valid

1 ☐(A ⊃ B)
2 u:A
* 3 ~R$\underline{u}$:B
[∴ (~$\underline{u}$:A • ~$\underline{u}$:B)
* 4 asm: ~(~$\underline{u}$:A • ~$\underline{u}$:B)
5 ∴ O~$\underline{u}$:B {from 3}
* 6 ∴ ~$\underline{u}$:B {from 5}
7 u ∴ ~B {from 7}
8 ∴ $\underline{u}$:A {from 4 and 6}
* 9 u ∴ (A ⊃ B) {from 1}
10 u ∴ ~A {from 7 and 9}
11 u ∴ A {from 8}
12 ∴ (~$\underline{u}$:A • ~$\underline{u}$:B) {from 4; 10 contradicts 11}

13.2a

2. The Cubs have a 12 percent chance of winning (60 • 20 percent).
4. The probability of six heads in a row is 1.5625 percent (50 • 50 • 50 • 50 • 50 • 50 percent).

6. Michigan has a 14.4 percent chance to win the Rose Bowl and a 85.6 percent (100 - 14.4 percent) chance to not win it. So the odds against Michigan winning it are 5.944 to 1 (unfavorable/favorable percent, or 85.6/14.4 percent). So you should win $59.44 (on a $10 bet) if Michigan wins it.
7. There are 16 cards worth 10 in the remaining 45 cards. So your chance of getting a card worth 10 is 35.6 percent [(16 • 100 percent)/45].
8. Here there are 32 cards worth 10 in the remaining 97 cards. So your chance of getting a card worth 10 is 33.0 percent [(32 • 100 percent)/97].
9. Your sister is being fair. Since 18 of the 36 combinations give an even number, the odds for this are even.
11. Since there are 4 5s in the remaining 47 cards, you have an 8.5 percent [(4 • 100)/47 percent] chance of getting a 5. Since there are 2 3s in the remaining 47 cards, you have a 4.3 percent [(2 • 100)/47 percent] chance of getting a 3.
12. If the casino takes no cut, it takes 2000 people to contribute a dollar to pay for someone winning $2000. So your chance of winning is 1 in 2000, or .05 percent. If the casino takes a large cut, your prospects could be much lower.
13. The probability is .27 percent (1/365).
14. We have a 30 percent chance of making the goal if we kick right now. Our chance is 35 percent (70 • 50 percent) if we try to make the first down and then kick. So we should go for the first down.

13.3a

2. You should believe it. It's 87.5 percent probable, since it happens in 7 of the 8 possible combinations.
4. You should believe it. It's 75 percent probable, since it happens in 6 of the 8 possible combinations.
6. Your expected gain is 10 percent with the bank, but 20 percent [(120 • 1) + (0 • 99) percent] with Mushy. If you want to maximize expected financial gain, you should go with Mushy. [To be safe, stay with the bank!]
7. The most that you'll agree to is $100. [You'll agree to more if you want to be sensible about risk-taking and see that the insurance company has to make a profit.]
8. The least that you'll agree to charge is $100 + expenses.
9. Your expected gain is 11 percent with Enormity, but only 10 percent [(.8 • 30) - (.2 • 70) percent] with Mushy. You should go with Enormity.

13.4a

2. The conclusion is too precisely stated. It's likely on the basis of this data that *roughly 80 percent* of all Loyola students were born in Illinois – but not that *exactly 78.4 percent* of them were.
4. The sample may be biased. Many of us associate mostly with people more or less like ourselves.
6. This weakens the argument. Since Lucy was sick and missed most of her classes, she's probably less prepared for this quiz.
7. This weakens the argument, since informal logic differs significantly from formal logic.
8. This strengthens the argument. In fact, it provides an independent (and stronger) argument for the conclusion.
9. This weakens the argument, since it raises the suspicion that Lucy might slacken off on the last quiz.
11. This example is difficult. Premise 2 may be the weakest premise; we've examined many orderly things (plants, spider webs, the solar system, etc.) that we *don't already know* to have intelligent designers – unless we presume the existence of God (which begs the question). Alvin Plantinga suggests that a better premise would be "Every orderly thing of which we know whether or not it has an intelligent designer in fact does have an intelligent designer." See his *God and Other Minds* for a discussion (and partial defense) of the argument.

13.5a

2. This strengthens the argument, since it points to increased points of similarity.
4. This doesn't affect the strength of the argument.
6. This doesn't affect the strength of the argument.
7. This strengthens the argument, since analogical reasoning is part of informal logic.
8. This doesn't affect the strength of the argument (unless we have further information on what logic books with cartoons are like).
9. This strengthens the argument, since analogical reasoning is part of inductive logic.
11. This strengthens the argument, since it increases the similarity between the two courses.
12. This one is tricky. Given no background information about utilitarianism, this item weakens the argument by pointing to a significant difference between the two courses. But, given the information that utilitarianism has to do with general ethical theory, the item strengthens the argument. If an applied ethics course treated this theory, then *even more so* a course in general ethical theory could be expected to cover it.
13. This strengthens the argument, since it increases the similarity between the two courses.
14. This doesn't affect the strength of the argument.

13.7a

2. Using the method of agreement, probably the combination of factors (having bacteria and food particles in your mouth) causes cavities, or else cavities cause the combination of factors. The latter is implausible (since it involves a present cavity causing a past combination of bacteria and food particles). So probably the combination of factors causes cavities.
4. By the method of variation, likely the variation in the time of the sunrise causes a variation in the time of the coldest temperature, or the second causes the first, or something else causes them both. The last two alternatives are implausible. So probably the variation in the time of the sunrise causes a variation in the time of the coldest temperature.
6. By the method of difference, probably the food I was eating is the cause (or part of the cause) of the invasion of the ants, or the invasion of the ants is the cause (or part of the cause) of my eating the food. The latter is implausible. So probably the food causes (or was part of the cause of) the invasion of the ants.
7. By the method of agreement, probably the presence of Megan causes the disappearance of the food, or else the disappearance of the food causes the presence of Megan. The latter is implausible. So probably the presence of Megan causes the disappearance of the food. (This all assumes something that the example doesn't explicitly state – namely, that no other factor correlates with the disappearance of the food.)
8. By the method of agreement, probably the presence of fluoride in the water causes a group to have less tooth decay, or else the fact that a group has less tooth decay causes the presence of fluoride in the water. The latter is implausible. So probably the presence of fluoride in the water causes a group to have less tooth decay.
9. By the method of difference, probably having fluoride in the water is the cause (or part of the cause) of the lower tooth decay rate, or else the lower tooth decay rate is the cause (or part of the cause) of the presence of fluoride in the water. Since *we* put fluoride in the water, we reject the second alternative. So probably having fluoride in the water is the cause (or part of the cause) of the lower tooth decay rate.
11. By the method of agreement, probably either Will's throwing food on the floor causes parental disapproval, or the disapproval causes Will to throw food on the floor. The second alternative is implausible. So probably Will's throwing food on the floor causes parental disapproval.

12. Mill's methods don't apply here. We can't conclude that marijuana causes heroin addiction. To apply the method of agreement, we'd have to know that people who use marijuana always or generally become heroine addicts.
13. By the method of variation, likely the rubbing is (or is part of) the cause of the heat, or the heat is (or is part of) the cause of the rubbing. The second alternative is implausible, since *we* cause the rubbing. So likely the rubbing is (or is part of) the cause of the heat.
14. By the method of variation, likely how long Alex studies is the cause of his grade, or his grade is the cause of how long he studies, or something else is the cause of both. If we are thinking about the *immediate* cause of the grade, the second alternative is implausible. (But getting good grades may encourage more studying later.) The third alternative could be true; maybe Alex is more interested in certain areas, and this interest causes more study and better grades in these areas. So likely how long Alex studies is [a major part of] the cause of his grade, or else something else is the cause of both.
16. By the method of variation, likely moving the lever causes the sound to vary, or the varying of the sound causes the lever to move, or something else is the cause of both. Since Will himself causes the lever to move, the second and third alternatives are implausible. So probably moving the lever causes the sound to vary.
17. By the method of agreement, probably aerobic exercise causes the lower heart rate, or the lower heart rate causes a person to do aerobic exercise. The second alternative is implausible, since the aerobic exercise comes first and then the lower heart rate later. So probably the aerobic exercise causes the lower heart rate.
18. By the method of agreement, probably the solidification from a liquid state causes the crystalline structure, or the crystalline structure causes the solidification from a liquid state. The second alternative is implausible, since the solidification comes about first, before the crystalline structure. So probably the solidification from a liquid state causes the crystalline structure.
19. By the method of agreement, probably night causes day or day causes night. However, in terms of our general knowledge, we know that neither alternative is correct. Some third combinations of factors, the light from the sun and the rotation of the earth, causes both day and night. (Mill's methods are rough guides and don't always work.)
21. By the method of agreement, probably Kurt's wearing the headband causes him to make the field goals, or his making the field goals causes him to wear the headband. The latter is implausible, since wearing the headband comes first and the field goals come later. So probably Kurt's wearing the headband causes him to make the field goals. [More ultimately, Kurt's *belief* in his lucky headband may cause him to make the field goals when he's wearing it and to miss the field goals when he isn't. To test this hypothesis using the method of difference, substitute another headband when Kurt isn't looking and then see whether he makes his field goals.]
22. By the method of difference, probably the water temperature was the cause (or part of the cause) of the death of the fish, or the death of the fish was the cause (or part of the cause) of the water temperature. The second alternative is implausible, since the temperature is controlled by the thermometer and doesn't change when fish die. So probably the water temperature was the cause (or part of the cause) of the death of the fish.
23. By the method of agreement, probably a combination of factors (getting exposed to the bacteria when having an average or low heart rate) causes the sickness and death, or the sickness and death causes this combination of factors. Since the factors come first, the second alternative is implausible. So probably the combination of factors causes the sickness and death.
24. By the method of variation, probably a higher inflation rate causes growth in the

national debt, or growth in the national debt causes a higher inflation rate, or something else causes them both.

13.8a

2. First, we'd need some way to identify germs (perhaps using a microscope) and colds. Then we'd verify that when one occurs then so does the other. From Mill's method of agreement, we'd conclude that either germs cause colds or else colds cause germs. We'd eliminate the second alternative by observing that if we first introduce germs then later we'll have a cold (while we can't do it the other way around). We'd further support the conclusion by showing that eliminating germs eliminates the cold.
4. We'd pick two groups which are alike as much as possible and have one regularly and moderately use alcohol and the other regularly and moderately use marijuana. We'd then note the effects. One problem is that the harmful effects might show up only after a long period of time; so our experiment might have to continue for many years. Another problem is that it might be difficult to find sufficiently similar groups who would abide by the terms of the experiment over a long period of time.
6. We'd study two groups of married women as alike as possible except that the first group is career oriented while the second is home oriented. Then we'd use surveys or interviews to try to rate the successfulness of the marriages. One problem is that there may be different views of when a marriage is "successful." Also, many might be mistaken in appraising the success of their marriage.
7. First, we'd need some way to identify factor K and cancer. Then we'd need to verify that when one occurs then so does the other. From Mill's method of agreement, we'd conclude that either factor K causes cancer or else cancer causes factor K. We'd eliminate the second alternative by observing that if we first introduce factor K (in an animal) then later we'll have cancer. We'd further support the conclusion by showing that eliminating factor K eliminates the cancer.
8. First, we'd need some way to identify hydrogen and oxygen and to know when we have a certain number of atoms of each. Then we'd try to find ways of converting water to hydrogen and oxygen – and hydrogen and oxygen back to water. We'd note that we get twice as many hydrogen atoms as oxygen atoms when we convert from water. Finally, we'd somehow have to eliminate the possibility that water contains 4 hydrogen atoms and 2 oxygen atoms (or 6 + 3, or 8 + 4, …).
9. The evidence for this would be indirect. We'd trace fossil remains and see whether they fit the patterns suggested by the theory. We'd study current plants and animals and see whether the theory explains their characteristics. We'd study the current growth and development in species – and try to produce new species of plants and animals. What is important here isn't a single "crucial experiment" (as in our Ohm vs. Mho case) but rather how the theory explains and unifies an enormous amount of biological data.

14.2a

2. "Filthy rich" is negative. "Affluent," "wealthy," and "rich" are more neutral. "Prosperous," "thriving," and "successful" are positive.
4. "Extremist" is negative. "Radical" and "revolutionary" are more neutral.
6. "Bastard" is negative. "Illegitimate," "fatherless," and "natural" are more neutral.
7. "Baloney" is negative. "Bologna" is neutral.
8. "Backward society" is negative. "Developing nation" and "third-world country" are neutral.
9. "Authoritarian" is negative. "Strict" and "firm" are neutral or positive.
11. "Hair-splitter" is negative. "Precise thinker," "exact thinker," and "careful reasoner" are positive.
12. "Egghead" is negative. "Thinker," "scholar," "intellectual," "learned," and "studious" are positive.

13. "Bizarre idea" is negative. "Unusual," "atypical," "uncommon," "different," and "unconventional" are neutral. "Imaginative," "extraordinary," "novel," and "innovative," are positive.
14. "Kid" is negative. "Youth," "youngster," and "young person" are neutral.
16. "Gay" is neutral or positive. "Fag" is negative.
17. "Abnormal" is negative. "Unusual," "atypical," "uncommon," "different," and "unconventional" are neutral.
18. "Bureaucracy" is negative. "Organization," "management," and "administration" are neutral.
19. "Abandoning" is negative. "Leaving," "departing," and "going away" are neutral.
21. "Brazen" is negative. "Bold," "fearless," "confident," and "unafraid" are neutral. "Brave," "courageous" and "daring" are positive.
22. "Old broad" is negative. "Mature woman" and "elderly lady" are neutral or positive.
23. "Old moneybags" is negative. "Affluent," "wealthy," and "rich" are more neutral. "Prosperous," "thriving," and "successful" are positive.
24. "Busybody" is negative. "Inquisitive," "interested," and "curious" are neutral.
26. "Old fashioned" is negative. "Traditional" and "conservative" are neutral.
27. "Brave" is positive. "Brazen," "foolhardy," "reckless," "rash," "careless," and "imprudent" are negative.
28. "Garbage" is negative. "Waste materials" and "refuse" are neutral.
29. "Cagey" is negative. "Clever," "shrewd," "astute," "keen," and "sharp" are neutral or positive.

14.3a

2. These exact ages are too precise for the vague "adolescent." "Between puberty and adulthood" would be better.
4. Subjects other than metaphysics may induce sleep. And many don't find metaphysics sleep-inducing.
6. Plucked chickens and apes are featherless bipeds but not human beings. In addition, one who took drugs to grow feathers wouldn't cease being a human being.
7. I believe many things (e.g. that I'll live at least ten years longer) that I don't know to be true.
8. A lucky guess (e.g. I guess right that the next card will be an ace) is a true belief but not knowledge.
9. This would make anything you sit on (the ground, a rock, your brother, etc.) into a chair.
11. "The earth is round" was true in 1000 B.C. (the earth hasn't changed shape!) but not proved. Also, the definition is circular – it defines "true" using "true."
12. Many invalid arguments persuade, and many valid ones (e.g. very complex ones or ones with absurd premises) don't persuade.
13. Killing in self-defense need not be murder.
14. Things against the law (e.g. protesting a totalitarian government) need not be wrong. And many wrong things (e.g. lying to your spouse) aren't against the law.

14.3b

2. This is false according to cultural relativism (CR).
4. This is false according to CR, since "This is good" on CR means "This is socially approved" – and the latter is true or false. However, we might understand statement 4 to mean that judgments about what is good aren't true or false *objectively* (i.e. independently of human opinions and feelings); statement 4, taken this way, is true according to CR.
6. This is true according to CR.
7. This is undecided.
8. This is true according to CR.
9. This is undecided by the definition. But almost all cultural relativists accept it.
11. This is true according to CR.
12. This is true according to CR.
13. This is false according to CR.
14. This is true according to CR.
16. This is undecided.
17. This is true according to CR.
18. This is false according to CR.

19. This (self-contradiction?) is true according to CR.

14.5a

2. This is meaningful on LP, since we could verify it by getting the correct time on the phone. It's also meaningful on PR, since its truth could make a practical difference regarding sensations (when we phone the time) and actions (if the clock is fast then maybe we should reset it or not depend on it).
4. This claim (from the Beatles's song "Strawberry Fields Forever") is probably meaningless on both criteria – unless it's given some special sense.
6. This is meaningless on both views. If everything doubled (including our rulers), we wouldn't notice the difference.
7. This is meaningless on both views. There isn't any observable or practical difference between wearing such a hat and wearing no hat at all.
8. This is meaningless on LP, since it couldn't be verified publicly. It's meaningful on PR, since its truth could make an experiential difference to Regina.
9. This is meaningless on LP, since it couldn't be verified publicly. It's meaningful on PR, since its falsity could make an experiential difference to others.
11. This is meaningless on LP, since it couldn't be verified publicly. It's meaningful on PR, since its truth could make an experiential difference to the angels.
12. This is meaningless on LP, since it couldn't be verified publicly. It's meaningful on PR, since its truth could make an experiential difference to God.
13. This is meaningless on LP, since it couldn't be verified publicly. It's meaningful on PR, since its truth could make a difference to how we ought to form our beliefs.
14. This (PR) seems to be meaningless on LP (since it doesn't seem able to be empirically tested). It is meaningful (and true) on PR, since its truth could make a difference in our choices regarding what we ought to believe.

14.6a

(These answers were adapted from those given by students)

2. "Is this unusual monkey a <u>rational</u> animal" could mean such things as:
- Is this unusual monkey sane (by whatever standards of sanity apply to monkeys)?
- Is this unusual monkey able to:
    - grasp general concepts (such as "monkey")?
    - grasp abstract concepts (such as "self-contradictory")?
    - reason (infer conclusions deductively from premises, weigh evidence in order to come to a conclusion, etc.)?
    - know itself as a knower (investigate the grounds of its knowledge, answer conceptual questions such as this one, pass a logic course, etc.)?
    - judge between right and wrong?
    - consider alternative actions it might perform, weigh the pros and cons of each, and make a decision based on this?
    - make the appropriate responses in sign languages that, if made by a human, would normally be taken to demonstrate the ability to grasp general concepts (or to do any of the other things mentioned above)?

4. "Are material objects <u>objective</u>?" could mean such things as:
- Are material objects distinct from our perception of them – so that they would continue to exist even if unperceived and even if there were no minds?
- Are different observers able by and large to agree on questions regarding the existence and properties of material objects – regardless of the varying backgrounds and feelings of the observers?
- Do material objects ("in themselves") really have the properties (colors, etc.) that we perceive them to have?

6. "Are scientific generalizations ever <u>certain</u>?" could be asking whether they are:
- logically necessary truths.
- self-evident or *a priori* truths.

- unchangeable and exceptionless over all periods of space and time.
- 100 percent probable.
- so firmly established that we can reasonably rule out ever having to modify them.
- so firmly established that we can reasonably rely on them for now.
- held without doubt in our own minds.

7. "Was the action of that monkey a free act?" could be asking whether this action was:

- one we didn't have to pay to watch.
- uncoerced (e.g. it wasn't pushed or threatened).
- self-caused (not influenced or compelled by external influences).
- to be explained by the monkey's goals and motives.
- not the result of a conditioning process or hypnosis.
- unpredictable (by causal laws).
- not necessary (so that given the exact same circumstances the monkey could have acted differently).
- the result of an uncoerced decision.
- the result of an uncoerced decision that in turn was not causally necessitated by prior circumstances (heredity, environment, etc.) beyond the control of the monkey?

8. "Is truth changeless?" could mean such things as:

- Are there statements without a specified time (e.g. "It's raining") that are true at one time but false at another?
- Are there statements without a specified time (e.g. "2+2=4") that are true at all times?
- Are there statements with a specified time and place (e.g. "The Chicago O'Hare Airport got 9 inches of rain on August 13–14, 1987") that are true at one time but can later become false?
- Do beliefs change?
- Are there some beliefs that are universally held by all people at all times?
- Are there degrees of being true?
- Is being true relative – so that we shouldn't ask "Is this true?" but only "Is this true for this person at this time?"?

9. "How are moral beliefs explainable?" could mean such things as

- By what basic moral principles can concrete moral judgments be justified and explained?
- By what means, if any, can basic moral principles be justified or proved (e.g. by appeal to self-evident truths, empirical facts, religious beliefs, etc.)?
- How can we explain why individuals or groups hold the moral beliefs they hold (or why they hold any moral beliefs at all)?
- How can we communicate moral beliefs?
- What does "moral belief" mean?
- How are "ought"-judgments related to "is"-judgments?

11. "Is the fetus a human being (or human person)?" could be asking whether it has:

- human parents.
- a human genetic structure.
- the ability to live apart from the mother (with or without support apparatus)?
- membership in the species *homo sapiens*.
- human physical features.
- human qualities of thought, feeling, and action.
- the capacity to develop human qualities of thought, feeling, and action.
- a strong right to live (and not to be killed).

12. "Are values objective?" could be asking whether some or all value judgments are:

- true or false.
- true or false independently of human beliefs and goals.
- universally shared.
- arrived at impartially.
- knowable through some rational method that would largely bring agreement among individuals who used the method correctly.
- truths that state that a given sort of action is always right (or wrong) regardless of circumstances and consequences.

13. "What is the nature of man?" could mean such things as (where in each of these we could take "man" as "human being" or as "adult *male* human being"):

- What does "human being" mean?
- What (or what most basically) distinguishes humans from other animals?

- How (or how most basically) can human beings be described (from the point of view of psychology, sociology, history, common sense, etc.)?
- What in humans is not a result of the influences of a given environment or society but rather is common to all humans?
- What was human life like prior to the creation of society?
- What is the metaphysical structure of the human person? (Is the human person a soul imprisoned in a body, or just a material body, or a composite of body and soul, or what?)
- What is the goal of human beings (as given by nature, God, evolution, etc.)?
- What is the origin and destiny of humanity?
- How ought humans to live?

14. "Can I ever <u>know</u> what someone else feels?" could mean such things as:
- Can I ever know (with reasonable evidence, or with absolute certitude) that another person has some specified feeling?
- Can we, from facts about observable behavior, deduce facts about the inner feelings of another?
- Can I ever know another's feelings in the immediate way that I know my own feelings?
- Can I ever vividly and accurately imagine what it would be like to have a given person's feelings (empathy)?
- Can I ever know that the inner experience labeled by another person as, for example, "fear" feels the same as the inner experience that I would label as "fear"?

16. "Is the world <u>illogical</u>?" could mean such things as:
- Does the world often surprise us and shatter our preconceptions?
- Are there many aspects of the world that cannot be rigidly systematized?
- Are people frequently ilogical (contradicting themselves or reasoning invalidly)?
- Are people often more easily moved by rhetoric and emotion than by logically correct reasoning?
- Can the premises of a valid argument be true while the conclusion was false?

14.7a

2. Analytic. (?)
4. Synthetic.
6. Analytic.
7. Synthetic.
8. Synthetic.
9. Analytic (by the definition of "100°C").
11. Synthetic. When black swan-like beings were discovered, people decided not to make "white" part of the definition of swan.
12. Analytic. (?)
13. Synthetic. (?)
14. Analytic.
16. Synthetic. (?)
17. Analytic.
18. Synthetic. (?)
19. Synthetic. (?)
21. Synthetic. (?)
22. Synthetic. (?)
23. Analytic.
24. Analytic.

14.8a

2. *A priori*. (?)
4. *A posteriori*.
6. *A priori*.
7. *A posteriori*.
8. *A posteriori*.
9. *A priori*.
11. *A posteriori*.
12. *A priori*. (?)
13. *A posteriori*. (?)
14. *A priori*.
16. *A posteriori*. (?)
17. *A priori*.
18. *A priori*. (?)
19. *A priori*. (?)
21. *A priori*. (?)
22. *A priori*. (?)
23. *A priori*.
24. *A priori*.

15.2a

2. Circular.

4. *Ad hominem*, false stereotype, or appeal to emotion.
6. Black-and-white thinking.
7. Beside the point (we have to show that the veto was the right move – not that it was decisive or courageous) or appeal to emotion (we praise the veto instead of giving reasons for thinking that it was the right move).
8. Division-composition.
9. *Ad hominem*.
11. Ambiguity. "Law" could mean "something legislated" (which requires a law-giver) or "observed regularity" (which doesn't so clearly seem to require this).
12. Beside the point (we have to show that Smith committed the crime – not that the crime was horrible) or appeal to emotion (we stir up people's emotions instead of showing that Smith committed the crime).
13. Circular.
14. Appeal to emotion or *ad hominem*.
16. Division-composition.
17. *Post hoc ergo propter hoc*.
18. Ambiguity. "Man" could mean "human being" or "adult male human being"; on either meaning, one or the other of the premises is false.
19. Appeal to force.
21. *Ad hominem*, false stereotype, or appeal to emotion.
22. Pro-con. What are the disadvantages of the proposal?
23. Complex question. This presumes "You're a good boy if and only if you go to bed now." (I prefer to ask, "Do you want to go to bed right now or in five minutes?")
24. Appeal to ignorance (unless it's a trial).
26. Straw man or false stereotype.
27. Division-composition.
28. False stereotype.
29. Appeal to ignorance.
31. *Ad hominem*.
32. Genetic fallacy.
33. *Post hoc ergo propter hoc*.
34. Complex question; it assumes that you killed the butler.
36. Circular.
37. Black-and-white thinking.
38. Appeal to the crowd.
39. This could be considered circular (if calling it un-American means that it ought to be opposed) or opposition (if it means that our opponents favor it) or appeal to emotion (if it's just derogatory language).
41. Ambiguity ("abnormal" shifts from "not typical" to "not healthy") or false stereotype.
42. Division-composition.
43. Circular.
44. Appeal to the crowd.
46. *Post hoc ergo propter hoc*.
47. Appeal to ignorance.
48. Appeal to emotion.
49. Straw man.

15.3a

(The answers for 2, 3, and 4 can have a different order – as can those for 5, 6, and 7.)

2. If we have ethical knowledge, then either ethical truths are provable or there are self-evident ethical truths.
   There are no self-evident ethical truths.
   Ethical truths aren't provable.
   ∴ We have no ethical knowledge.

4. If we have ethical knowledge, then either ethical truths are provable or there are self-evident ethical truths.
   We have ethical knowledge.
   There are no self-evident ethical truths.
   ∴ Ethical truths are provable.

6. All human concepts derive from sense experience.
   The concept of logical validity doesn't derive from sense experience.
   ∴ The concept of logical validity isn't a human concept.

7. The concept of logical validity is a human concept.
   The concept of logical validity doesn't derive from sense experience.
   ∴ Not all human concepts derive from sense experience.

8. Yes, if an argument is valid then its turnaround also is valid. Consider argument "A, B ∴ C" and its turnarounds "A, not-C ∴ not-B" and "not-C, B ∴ not-A." Each is

valid if and only if the set "A, B, not-C" is inconsistent. So if any of the three is valid, then all three are.

9. If "No statement is true" is true, then some statement is true. Statement 9 implies its own falsity and hence is self-refuting.
11. This statement hasn't been proved. So on its own grounds we shouldn't accept it.
12. We can't decide the truth or falsity of this statement through scientific experiments. So on its own grounds it's meaningless.
13. Then this claim itself isn't true.
14. This itself hasn't been proved using experimental science. So on its own grounds we cannot know this statement.

15.4a

(These are examples of answers and aren't the only "right answers.")

2. Genocide in Nazi Germany was legal.
   Genocide in Nazi Germany wasn't right.
   ∴ It's false that every act is right if and only if it's legal.
4. If the agent and company will probably get caught, then offering the bribe probably doesn't maximize the long-term interests of everyone concerned.
   The agent and company will probably get caught. (One might offer an inductive argument for this one.)
   ∴ Offering the bribe probably doesn't maximize the long-term interests of everyone concerned.

   (Also, one might appeal to the premise that replacing open and fair competition with bribery will bring about inferior and expensive products – which isn't in the public interest.)
6. Prescribing this medicine was a wrong action (as it turned out – because the patient was allergic to it).
   Prescribing this medicine was an error made in good faith (the doctor was trying to do the best she could – and she had no way to know about the patient's allergy).
   ∴ Some wrong actions are errors made in good faith.
7. All blameworthy actions are actions where the agent lacks the proper motivation to find out and do what is right.
   No error made in good faith is an action where the agent lacks the proper motivation to find out and do what is right.
   ∴ No error made in good faith is blameworthy.
8. Some acts of breaking deep confidences are socially useful.
   No acts of breaking deep confidences are right.
   ∴ Not all socially useful acts are right.
9. Some acts of punishing the innocent in a minor way to avert a great disaster are right.
   All acts of punishing the innocent in a minor way to avert a great disaster are acts of punishing the innocent.
   ∴ Some acts of punishing the innocent are right.
11. "All beliefs unnecessary to explain our experience ought to be rejected" is a self-refuting statement (since it too is unnecessary to explain our experience and so ought to be rejected on its own grounds).
    All self-refuting statements ought to be rejected.
    ∴ "All beliefs unnecessary to explain our experience ought to be rejected" ought to be rejected.
12. If "All beliefs which give practical life benefits are justifiable pragmatically" isn't true, then there is no justification for believing in the reliability of our senses and rejecting skepticism about the external world.
    There is justification for believing in the reliability of our senses and rejecting skepticism about the external world.
    ∴ "All beliefs which give practical life benefits are justifiable pragmatically" is true.
13. The idea of a perfect circle is an idea that we use in geometry.
    All ideas that we use in geometry are human concepts.

∴ The idea of a perfect circle is a human concept.

14. If the idea of a perfect circle derives from sense experience, then we've experienced perfect circles through our senses.
    We haven't experienced perfect circles through our senses.
    ∴ The idea of a perfect circle doesn't derive from sense experience.